On the Control of Cooperative Robots Without Velocity Measurements

Jorge Gudiño-Lau, Marco A. Arteaga, Luis A. Muñoz, and Vicente Parra-Vega

Abstract—One of the main practical problems on dexterous robots is the complexity of integrating a large amount of sensors within a small robot architecture. In this brief, the control of cooperative robots, without using velocity measurements, is considered. Our main purpose is to analyze the feasibility of avoiding velocity measurements to manipulate an object firmly. Experimental results are shown to support the developed theory.

Index Terms—Linear observers, robust control.

I. INTRODUCTION

 Dexterity is one of the most desirable behaviors a robot should be asked to have. This property can be achieved through a robot hand with a combination of good performance between position and force control. Thus, robot hands (as well as cooperative robots), may find many areas of application today [1]. For example, today’s industrial robots are characterized by a limited number of specific applications, so that “traditional robotics” requires extending actual industrial robot capabilities. Also, there is a constant interest toward prosthetic devices for humans who have lost their limb. Finally, dexterous telerobtics is an actual desirable technology application.

Early attempts to establish a relationship between the automatic control of robots carrying out a shared task are referred to Kathib’s operational space formulation [2]. During the 1980s, the most important research results considered the contact evolution during manipulation [3]. Such a contact evolution requires a perfect combination of position and force control. Some of the first approaches following the objective of combining position and force control are presented in [4] and [5]. In those works, the dynamics of the object is considered explicitly. In [6]–[8], control schemes which do not take into account the dynamics of the object but rather the motion constraints are designed. These control approaches have the advantage that they do not require an exact knowledge of the system model parameters, since an adaptive approach is introduced. More recently, Shlegl et al. [9] show some advances on hybrid (in terms of a combination of continuous and discrete systems) control approaches. Despite the fact that Mason [10] proposed the base of sensorless manipulation in the 1980s, there are few control algorithms for cooperative robot systems which take into account the possible lack of velocity measurements. It is worthy noticing that, since a digital computer is usually employed to implement a control law, a good approximation of the velocity vector can be obtained by means of numerical differentiation. However, recent experimental results have shown that a (digitalized) observer in a control law performs better [11]. For this reason, this brief presents a decentralized control algorithm for cooperative robots (or robot hands) which achieves asymptotic stability of tracking of desired positions and forces by using a nonlinear observer.

The brief is organized as follows. In Section II, the system model and its properties are presented. Section III gives the proposed control and observer law, while Section IV shows experimental results. Conclusions are drawn in Section V.

II. SYSTEM MODEL AND PROPERTIES

Consider a cooperative system with \( I \)-fingers, each of them with \( n_i \) degrees of freedom and \( m_ic \) constraints arising from the contact with a held object. Then, the total number of degrees of freedom is given by \( n = \sum_{i=1}^{I} n_i \) with a total number of \( m = \sum_{i=1}^{I} m_i \) constraints, with \( n_i > m_i \). The dynamics of the \( i \)th finger is given by [8]

\[
H_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + D_i \dot{q}_i + g_i(q_i) = \tau_i + J_i^T(\varphi_i) \lambda_i \quad (1)
\]

where \( q_i \in \mathbb{R}^{n_i} \) is the vector of generalized joint coordinates, \( H_i(q_i) \in \mathbb{R}^{n_i \times n_i} \) is the symmetric positive definite inertia matrix, \( C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n_i} \) is the vector of Coriolis and centrifugal torques, \( D_i(\dot{q}_i) \in \mathbb{R}^{m_i} \) is the vector of gravitational torques, \( g_i(q_i) \in \mathbb{R}^{m_i} \) is the positive semidefinite diagonal matrix accounting for joint viscous friction coefficients, \( \tau_i \in \mathbb{R}^{m_i} \) is the vector of torques acting at the joints, and \( \lambda_i \in \mathbb{R}^{m_i} \) is the vector of Lagrange multipliers (physically represents the force applied at the contact point). \( J_i(\varphi_i) = \nabla \varphi_i(q_i) \in \mathbb{R}^{m_i \times n_i} \) is assumed to be full rank in this brief. \( \nabla \varphi_i(q) \) denotes the gradient of the object surface vector \( \varphi_i \in \mathbb{R}^{m_i} \) which maps a vector onto the normal plane at the tangent plane that arises at the contact point described by

\[
\varphi_i(q_i) = 0. \quad (2)
\]

Note that (2) means that homogeneous constraints are being considered [8]. The complete system is subjected to \( m \) holonomic constraints given by

\[
\varphi(q) = 0 \quad (3)
\]

where \( \varphi(q) = \varphi(q_1, \ldots, q_I) \in \mathbb{R}^m \). This means that the object being manipulated and the environment are modeled by the constraint (3).

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Let us denote the largest (smallest) eigenvalue of a matrix by \( \lambda_{\text{max}}(\cdot) (\lambda_{\text{min}}(\cdot)) \). The norm of an \( n \times 1 \) vector \( x \) is defined by the formula \( \|x\| \triangleq \sqrt{x^T x} \), while the norm of an \( m \times n \) matrix \( A \) is the corresponding induced norm \( \|A\| \triangleq \sqrt{\lambda_{\text{max}}(A^T A)} \).

By recalling that revolute joints are considered, the following properties can be established [7], [8], [12]:

**Property 2.1:** Each \( H_i(q_i) \) satisfies \( \lambda_{\text{hi}} \|x\|^2 \leq x^T H_i x \leq \lambda_{\text{hi}} \|x\|^2 \) \( \forall x, q_i \in \mathbb{R}^{n_i} \), where \( \lambda_{\text{hi}} \triangleq \min_{q_i \in \mathbb{R}^{n_i}} \lambda_{\text{min}}(H_i) \), \( \lambda_{\text{hi}} \triangleq \max_{q_i \in \mathbb{R}^{n_i}} \lambda_{\text{max}}(H_i) \), and \( 0 < \lambda_{\text{hi}} < \infty \).

**Property 2.2:** With a proper definition of \( C_i(q_i, \dot{q}_i) \) and \( \bar{H}_i(q_i) = 2C_i(q_i, \dot{q}_i) \) is skew-symmetric.

**Property 2.3:** The vector \( C_i(q_i, \dot{q}_i) y \) satisfies \( C_i(q_i, \dot{q}_i) y = C_i(q_i, y) x \forall x, y \in \mathbb{R}^{n_i} \).

**Property 2.4:** It is satisfied \( \|C_i(q_i, \dot{q}_i)\| \leq k_{\text{ci}} \|x\| \) with \( 0 < k_{\text{ci}} < \infty \) \( \forall x \in \mathbb{R}^{n_i} \).

**Property 2.5:** The vector \( \dot{q}_i \) can be written as

\[
\dot{q}_i = q_i + (J_{\dot{q}_i}^+ \dot{q}_i - J_{\dot{q}_i}^+ J_{\dot{q}_i} \dot{q}_i) = \underbrace{(I_{n_i} \times n_i)}_{\Delta} \dot{q}_i + (J_{\dot{q}_i}^+ \dot{q}_i - J_{\dot{q}_i}^+ J_{\dot{q}_i} \dot{q}_i) \triangleq \dot{p}_i + J_{\dot{q}_i}^+ \dot{q}_i - J_{\dot{q}_i}^+ J_{\dot{q}_i} \dot{q}_i = Q_i(q_i, \dot{q}_i) \dot{p}_i \]

where \( J_{\dot{q}_i}^+ = (J_{\dot{q}_i} J_{\dot{q}_i}^T)^{-1} J_{\dot{q}_i} \in \mathbb{R}^{n_i \times n_i} \) stands for the Penrose's pseudoinverse and \( Q_i \in \mathbb{R}^{n_i \times n_i} \) satisfies \( \text{rank}(Q_i) = n_i - m_i \). These two matrices are orthogonal, i.e., \( Q_i^T J_{\dot{q}_i} = 0 \) and \( Q_i J_{\dot{q}_i}^T = 0 \). \( \dot{p}_i \triangleq J_{\dot{q}_i} \dot{q}_i \in \mathbb{R}^{n_i} \) is the so-called constrained velocity. Furthermore, in view of constraint (3), it holds

\[
\sum_{i=1}^{l} \dot{p}_i = 0 \quad \text{and} \quad \sum_{i=1}^{l} p_i = \sum_{i=1}^{l} \int_0^t J_{\dot{q}_i} \dot{q}_i \text{d}t = 0.
\]

Since homogeneous constraints are being considered, it also holds in view of (2) that

\[
p_i = 0 \quad \text{and} \quad \dot{p}_i = 0
\]

for \( i = 1, \ldots, l \). \( p_i \) is called the constrained position.

To be able to design the control-observer scheme, the following assumptions are made.

**Assumption 2.1:** The \( l \) robots of which the system is made up satisfy constraints (2) and (6) for all time. Furthermore, none of the robots is redundant.

**Assumption 2.2:** The matrix \( J_{\dot{q}_i} \) is Lipschitz continuous, i.e.,

\[
\|J_{\dot{q}_i}(q_i) - J_{\dot{q}_i}(q_{i\text{d}})\| \leq L_i \|q_i - q_{i\text{d}}\|
\]

for a positive constant \( L_i \) and for all \( q_i, q_{i\text{d}} \in \mathbb{R}^{n_i} \). Besides, there exists a positive–finite constant \( C_{\text{di}} \) which satisfies

\[
C_{\text{di}} = \max_{q_i \in \mathbb{R}^{n_i}} \|J_{\dot{q}_i}^+(q_i)\|.
\]

**A. Control Law**

In this section, the control problem of a cooperative system of rigid robots is studied. Consider model (1) and define the tracking and observation errors as

\[
\dot{\tilde{q}}_i = q_i - q_{i\text{d}} \triangleq \tilde{q}_i
\]

where \( q_{i\text{d}} \) is a desired smooth bounded trajectory satisfying constraint (2), and \((\cdot)\) represents the estimated value of \((\cdot)\). Other error definitions are

\[
\Delta p_i \triangleq p_i - p_{i\text{d}} \quad \Delta \lambda_i \triangleq \lambda_i - \lambda_{i\text{d}}
\]

where \( p_{i\text{d}} \) is the desired constrained position which satisfies (6). \( \lambda_{i\text{d}} \) is the force to be applied by each finger on the constrained surface. Other useful definitions are

\[
\dot{\tilde{q}}_i = Q_i(q_i) (\tilde{q}_i - \lambda_i (\dot{q}_i - q_{i\text{d}})) + J_{\dot{q}_i}^+(q_i) (\dot{p}_i - \beta_i \Delta p_i + \xi_i \Delta F_i)
\]

\[
s_i = Q_i(q_i) (\dot{q}_i - \lambda_i (q_i - q_{i\text{d}})) + J_{\dot{q}_i}^+(q_i) (\Delta p_i + \beta_i \Delta p_i - \xi_i \Delta F_i)
\]

\[
\Delta F_i \triangleq \int_0^t \Delta \lambda_i (\vartheta(t) \chi(t)) \text{d}t
\]

where \( \lambda_i = k_i I \in \mathbb{R}^{n_i \times n_i} \) with \( k_i > 0 \), and \( \xi_i \in \mathbb{R}^{n_i \times n_i} \) are diagonal positive–definite matrices, and \( \beta_i \) is a positive constant. Note that \( s_{\text{pi}} \) and \( s_{\text{fii}} \) are orthogonal vectors, and that \( s_i \) can also be written as

\[
s_i = Q_i(q_i) (\dot{q}_i + \lambda_i \ddot{q}_i - \lambda_i z_i) + J_{\dot{q}_i}^+(q_i) (\Delta p_i + \beta_i \Delta p_i - \xi_i \Delta F_i).
\]

Let us analyze \( \dot{\tilde{q}}_i \). This quantity is given by

\[
\dot{\tilde{q}}_i \triangleq Q_i(q_i) (\tilde{q}_i - \lambda_i (q_i - q_{i\text{d}})) + J_{\dot{q}_i}^+(q_i) (\dot{p}_i - \beta_i \Delta p_i + \xi_i \Delta F_i)
\]

\[
+ Q_i(q_i) (\tilde{q}_i - \lambda_i (q_i - q_{i\text{d}})) + J_{\dot{q}_i}^+(q_i) (\beta_i (\dot{p}_i - \tilde{p}_i) + \xi_i \Delta \lambda_i).
\]

As it will be shown later, \( \tilde{q}_i \) is necessary to implement the controller and the observer. However, this quantity is not available since \( \dot{q}_i \) is not measurable. In order to overcome this drawback, let us consider \( Q_i(q_i) \in \mathbb{R}^{n_i \times n_i} \). You have

\[
\dot{Q}_i(q_i) = \frac{\partial q_{i11}(q_i)}{\partial q_i} \dot{q}_i \quad \cdots \quad \frac{\partial q_{i1m}(q_i)}{\partial q_i} \dot{q}_i \\
\vdots \\
\frac{\partial q_{im1}(q_i)}{\partial q_i} \dot{q}_i \quad \cdots \quad \frac{\partial q_{imm}(q_i)}{\partial q_i} \dot{q}_i
\]
where \( a_{\alpha \beta} \) is the \( \alpha \beta \) element of \( Q(q_i) \). Based on (18), consider the following definition:

\[
\dot{Q}(q_i) \triangleq \begin{bmatrix}
\frac{\partial q_{11}(q_i)}{\partial q_i} \dot{q}_{i1} & \cdots & \frac{\partial q_{1n}(q_i)}{\partial q_i} \dot{q}_{in} \\
\vdots & \ddots & \vdots \\
\frac{\partial q_{n1}(q_i)}{\partial q_i} \dot{q}_{i1} & \cdots & \frac{\partial q_{nn}(q_i)}{\partial q_i} \dot{q}_{in}
\end{bmatrix}
\]

(19)

with

\[
\dot{q}_{i\alpha} \triangleq \dot{q}_{i\alpha} - \Lambda_i z_{i\alpha}.
\]

(20)

Then, one can compute

\[
\dot{Q}(r_i) = \begin{bmatrix}
\frac{\partial q_{11}(q_i)}{\partial q_i} \dot{r}_i & \cdots & \frac{\partial q_{1n}(q_i)}{\partial q_i} \dot{r}_i \\
\vdots & \ddots & \vdots \\
\frac{\partial q_{n1}(q_i)}{\partial q_i} \dot{r}_i & \cdots & \frac{\partial q_{nn}(q_i)}{\partial q_i} \dot{r}_i
\end{bmatrix}
\]

(21)

with \( \dot{Q}(r_i) \triangleq \dot{Q}(q_i) - \dot{Q}(\tilde{q}_{i\alpha}) \), and

\[
r_i \triangleq Q_i - \tilde{Q}_{i\alpha} = \dot{z}_i + \Lambda_i z_i.
\]

(22)

From (19), the following substitution for \( \tilde{q}_{i\alpha} \) is proposed.

\[
\tilde{q}_{i\alpha} \triangleq \dot{Q}(q_i) (\tilde{q}_{i\alpha} - \Lambda_i (\dot{q}_i - q_{i\alpha})) + J_{\phi_i} (q_i) (\tilde{q}_{i\alpha} - \dot{q}_i) - \beta_i \Delta p_i + \xi_i \Delta F_i + C_i(q_i, \dot{q}_i) s_i - K_{DRi} s_i.
\]

(23)

where \( J_{\phi_i}^{+}(q_i) \) is defined in the very same fashion as \( \dot{Q}(q_i) \) in (19). Note that \( \dot{p}_i \) is still used since this value is known from (6). After some manipulation, it is possible to get

\[
\tilde{q}_{i\alpha} = \tilde{q}_{i\alpha} + e_i (r_i).
\]

(24)

where

\[
e_i (r_i) \triangleq -\dot{Q}(r_i) (\dot{q}_{i\alpha} - \Lambda_i (\dot{q}_i - q_{i\alpha})) - J_{\phi_i} (r_i) (\dot{p}_{i\alpha} - \beta_i \Delta p_i + \xi_i \Delta F_i).
\]

(25)

The proposed controller is given for each single input by

\[
\tau_i \triangleq H_i(q_i) \tilde{q}_{i\alpha} + C_i(q_i, \dot{q}_i) \tilde{q}_{i\alpha} + D_i \dot{q}_{i\alpha} + g_i - K_{Pi} (s_i - r_i) - J_{\phi_i}^{+} (q_i) (\Lambda_i - k_{Fi} \Delta F_i)
\]

(26)

where \( K_{Pi} \in \mathbb{R}^{n_i \times n_i} \) is a diagonal positive–definite matrix and \( k_{Fi} \) is a positive constant. Note that from (14) and (22) it is \( \dot{q}_{i\alpha} = \dot{r}_i = s_i - r_i \). Thus, from (24) one gets

\[
\tau_i = H_i(q_i) \tilde{q}_{i\alpha} + e_i + C_i(q_i, \dot{q}_i) \tilde{q}_{i\alpha} + D_i \dot{q}_{i\alpha} + g_i - K_{Pi} (s_i - r_i) - J_{\phi_i}^{+} (q_i) (\Lambda_i - k_{Fi} \Delta F_i).
\]

(27)

By substituting (27) into (1), the closed-loop dynamics becomes

\[
H_i(q_i) \dot{s}_i = -C_i(q_i, \dot{q}_i) s_i - K_{DRi} s_i + K_{Pi} r_i + J_{\phi_i}^{+} (q_i) (\Lambda_i - k_{Fi} \Delta F_i)
\]

(28)

after some manipulation, where \( K_{DRi} \triangleq K_{Pi} + D_i \). In order to get (28), Property 2.3 has been used.

B. Observer Definition

The proposed dynamics of the observer is given by

\[
\dot{q}_{i\alpha} = \dot{q}_{i\alpha} + \Lambda_i z_i + k_{Di} z_i
\]

(29)

\[
\dot{q}_{i\alpha} = \dot{q}_{i\alpha} + k_{Di} \Lambda_i z_i + H_i^{-1} J_{\phi_i}^{+} (\Delta \lambda_i + k_{Fi} \Delta F_i)
\]

(30)

where \( k_{Di} \) is a positive constant. Since from (29) you have \( \dot{q}_{i\alpha} = \dot{q}_{i\alpha} + k_{Di} \Lambda_i z_i + H_i^{-1} J_{\phi_i}^{+} (\Delta \lambda_i + k_{Fi} \Delta F_i) \), (30) becomes

\[
\dot{s}_i = \dot{r}_i + k_{Di} r_i + e_i + H_i^{-1} J_{\phi_i}^{+} (\Delta \lambda_i + k_{Fi} \Delta F_i)
\]

(31)

in view of (24). By multiplying both sides of (31) by \( H_i(q_i) \) and by taking into account (28) one gets

\[
H_i(q_i) \dot{r}_i = -H_{Pi} r_i - C_i(q_i, \dot{q}_i) s_i - C_i(q_i, \dot{q}_i) s_i - K_{DRi} s_i
\]

(32)

where \( H_{Pi} \triangleq k_{Di} H_i(q_i) - K_{Pi} \). Finally, by using Property 2.3 again and some manipulation, it is

\[
H_i(q_i) \dot{r}_i = -C_i(q_i, \dot{q}_i) r_i - H_{Pi} r_i - K_{DRi} s_i - C_i(q_i, \dot{q}_i) s_i + \dot{q}_{i\alpha} \dot{r}_i.
\]

(33)

Now, let us define

\[
x_i \triangleq [s_i^T \ r_i^T \ \Delta F_i]^T
\]

(34)

as state for (15), (28), and (33). The main idea of the control–observer design is to show that whenever \( \|x_i\| \) tends to zero, the tracking errors \( \tilde{q}_{i\alpha}, \tilde{q}_{i\alpha}, \Delta p_i, \Delta \dot{p}_i, \) and \( \Delta \lambda_i \) and the observation errors \( z_i \) and \( \dot{z}_i \) will tend all to zero as well. From (22), it is rather obvious that if \( r_i \) is bounded and tends to zero, so will do \( z_i \) and \( \dot{z}_i \). But this is not clear for the other variables. The following lemma shows that this is indeed the case under some conditions.

**Lemma 3.1:** If \( x_i \) is bounded by \( x_{\text{max}} \), and tends to zero, then the following facts hold:

a) \( \Delta p_i \) and \( \Delta \dot{p}_i \) remain bounded and tend to zero;

b) \( \tilde{q}_{i\alpha} \) and \( \tilde{q}_{i\alpha} \) remain bounded. If the bound \( x_{\text{max}} \) for \( \|x_i\| \) is chosen small enough so as to guarantee that \( \|\tilde{q}_{i\alpha}\| \leq \eta_i \) for all \( t \), \( \eta_i \) a positive and small enough constant, then both \( \tilde{q}_{i\alpha} \) and \( \tilde{q}_{i\alpha} \) will tend to zero as well; and

c) If, in addition, the velocity vector \( \dot{q}_{i\alpha} \) is bounded, then \( \Delta \lambda_i \) will remain bounded and tend to zero.

The proof of Lemma 3.1 can be found in Appendix I. The main conclusion is that if \( x_i \) is bounded, so are \( s_i, r_i, \) and \( \Delta F_i \). Furthermore, they all tend to zero if \( z_i \) does. In fact, any other error signal will be bounded and tend to zero as well.

It is interesting to note that, if \( \|x_i\| \) is bounded by \( x_{\text{max}} \), then it is always possible to find a bound for \( e_i (r_i) \) in (25) which satisfies

\[
\|e_i (r_i)\| \leq M_{ei} (x_{\text{max}}) \|r_i\| < \infty.
\]

(35)
Consider the following Lyapunov candidate function for system (15), (28), and (33)

\[ V_i(x_i) = \frac{1}{2} x_i^T M_i x_i \]  

(36)

where \( M_i \) is block \( \text{diag}[H_i(q_i), H_i(q_i), \xi_i] \), and \( V_i(x_i) \) satisfies

\[ \lambda_{1i} \| x_i \|^2 \leq V_i(x_i) \leq \lambda_{2i} \| x_i \|^2 \]  

(37)

with

\[ \lambda_{1i} \triangleq \frac{1}{2} \min_{q_i \in \mathcal{F}_i} \lambda_{\min}(M_i) \]

\[ \lambda_{2i} \triangleq \frac{1}{2} \max_{q_i \in \mathcal{F}_i} \lambda_{\max}(M_i) \]  

(38)

for \( i = 1, \ldots, l \). Suppose you may find a region of attraction

\[ S_{\alpha i} = \{ x_i : \| x_i \| \leq a_i \} \]  

(39)

so that the derivative of \( \dot{V}_i(x_i) \leq 0 \) with \( V_i(x_i) = 0 \) if and only if \( x_i = 0 \). If \( a_i \) is chosen so that \( \| x_i \| \leq x_{\text{max}} \) is valid for all time and \( x_{\text{max}} \) is small enough in the sense of Lemma 3.1, then from the former discussion one can conclude the convergence to zero of all error signals. The following theorem establishes the conditions for the controller-observer parameters to guarantee this.

**Theorem 3.1:** Consider the cooperative system dynamics given by (1) and (2), in closed-loop with the control law (26) and the observer (29)–(30), where \( q_{\text{di}}, \bar{p}_{\text{di}} \) are the desired bounded-joint and constrained positions, whose derivatives \( \dot{q}_{\text{di}}, \ddot{q}_{\text{di}}, \bar{p}_{\text{di}}, \ddot{p}_{\text{di}} \) are also bounded, and they all satisfy constraint (6). Consider also (38), and \( l \) given regions of attraction defined by (39) for each subsystem, where the bounds \( a_i, i = 1, \ldots, l \), are chosen according to

\[ a_i \leq \frac{\eta_i \alpha_i}{(1 + \lambda_{\text{max}}(\xi_i) C_{0i} + \sqrt{m}) \sqrt{\lambda_{1i}}} \]  

(40)

with \( \alpha_i \) defined in Appendix I. Then, every dynamic and error signal remains bounded and asymptotic stability of tracking, observation and force errors arise, i.e.,

\[ \lim_{t \to \infty} \dot{q}_i = 0 \quad \lim_{t \to \infty} \ddot{q}_i = 0 \quad \lim_{t \to \infty} z_i = 0 \quad \lim_{t \to \infty} \Delta \lambda_i = 0 \]  

(41)

if the following conditions are satisfied:

\[ \lambda_{\min}(K_{Ri}) = \mu_{\Delta i} + 1 + \delta_{\Delta i} \]

\[ \delta_{\Delta i} = \frac{\lambda_{\text{max}}(K_{Ri}) + \omega_i}{\lambda_{\text{max}}(\xi_i)} \]  

(42)

\[ \omega_i = \mu_{\Delta i} + (1/4) (\lambda_{\Delta i} + \mu_{\Delta i} + \mu_{\Delta i}^2 + \delta_{\Delta i}) \]  

(43)

where \( \omega_i = \mu_{\Delta i} + (1/4) (\lambda_{\Delta i} + \mu_{\Delta i} + \mu_{\Delta i}^2 + \delta_{\Delta i}) \) \( (\Delta i) \) were positive constant and

\[ \mu_{\Delta i} \triangleq \max_{\| x_i \| \leq x_{\text{max}}} \| C_i(q_i, \dot{q}_i, \ddot{q}_i) \| \]  

(44)

\[ \mu_{\Delta i} \triangleq \max_{\| x_i \| \leq x_{\text{max}}} \| C_i(q_i, s_i, \dot{q}_i) \| \]  

(45)

where (35) and Properties 2.1 and 2.4 have been used.

\[ \mu_{\Delta i} \triangleq \max_{\| x_i \| \leq x_{\text{max}}} \| C_i(q_i, s_i, 2 \dot{q}_i) \| \]  

(46)

\[ \mu_{\Delta i} \triangleq M_{\Delta i}(x_{\text{max}}) \lambda_{\Delta i} \]  

(47)

\[ \lambda_{\Delta i} \triangleq \lambda_{\text{max}}(\dot{D}_i) \]  

(48)

\[ x_{\text{max}} = a_i \sqrt{\frac{\lambda_{\Delta i}}{\lambda_{\text{Di}}}} \]  

(49)

The proof of the Theorem 3.1 can be found in Appendix II.

**Remark 3.1:** The result of Theorem 3.1 is only local. This to guarantee the convergence to zero of the tracking errors \( \dot{q}_i \) and \( \ddot{q}_i \). However, this does not represent a serious drawback since for grasping it is usual to give smooth trajectories with zero initial position errors. On the other hand, it is worth pointing out that a controller-observer scheme is implemented for each robot separately, while only the knowledge of every restriction of the form (2) is required.

**IV. EXPERIMENTAL RESULTS**

In this section, some experimental results are presented. To this end, a test bed with two industrial robots is used (Fig. 1). The robots are at the Laboratory for Robotics of the National University of Mexico. They are the A465 and A255 of CRS Robotics. Even though the first one has six degrees of freedom and the second one five, only the first three joints of each robot are used for the experiments, while the rest of them are mechanically braked. Each joint is actuated by a CD motor. Thus, in order to implement control law (26) and observer (29)–(30), the motors dynamics has to be taken into account. Furthermore, as it may be appreciated in Fig. 1, only the robot A465 has a force sensor. For this reason, the trajectories to be followed must be relative simple, so that the single measurements of the force sensor can be used for the two manipulators.

The palm frame of the whole system is at the base of the robot A465, with its \( x \) axis pointing toward the other manipulator. If the task consists in lifting the object and pushing with a desired force, then the constraints in Cartesian coordinates are simply given by

\[ \varphi_i = x_i - b_i = 0 \]  

(50)

for \( i = 1, 2 \) and \( b_i \) a positive constant. The desired trajectories are given

\[ x_{d1} = 0.626 \text{ [m]} \quad x_{d2} = 0.936 \text{ [m]} \]  

(51)

\[ y_{d1,2} = 0.05 \sin(\omega (t - t_i)) \text{ [m]} \]  

(52)

\[ z_{d1,2} = (0.585 + 0.05 \cos(\omega (t - t_i)) - 0.05) \text{ [m]} \]  

(53)

Note that the inverse kinematics of the manipulators has to be employed to compute \( q_{\text{di}} \) for \( i = 1, 2 \). These trajectories are valid from an initial time \( t_i \) to a final time \( t_f \), while \( \omega \) is a fifth-order polynomial designed to satisfy \( \omega(t_i) = \omega(t_f) = 0 \). The derivatives of \( \omega \) are zero at \( t_i \) and \( t_f \). By choosing (51)–(53), the robots will make a circle in the \( y-z \) plane. The only difference between the trajectories for robots A465 and A255 is the width of the object. Also, no force control is carried out until the manipulators are in the initial position for the circle, at (0.626, 0, 0.585) [m] for the first manipulator
and \((0.936, 0, 0.585)\) [m] for the second one. The desired pushing force is given by
\[
f_{\text{dr}1,2} = 15 + 5 \sin(2\pi(t - t_1)/40) \ [N]
\]
and \(f_{\text{dy}1,2} = f_{\text{dz}1,2} = 0 \ [N]\). Note that in view of the desired trajectories, the force read by the single sensor can be used for both robots. The different control and observer parameters are \(\Lambda_1 = 12I, \Lambda_2 = 23I, K_{R1} = \text{diag}\{45, 60, 60\}, K_{R2} = \text{diag}\{15, 13, 13\}, k_{\text{d}1} = k_{\text{d}2} = 4, k_{\xi_1} = k_{\xi_2} = 5, \xi_1 = \xi_2 = 0.001I\).

The observer-controller scheme has been programmed in a PC computer, while the sampling time is \(9\) ms. The experiment lasts \(70\) s. The object is held at \(t = 7.5\) s. Before, the robots are in free movement. The control law (26) and the observer (29)–(30) are used with the force part set to zero (i.e., \(Q_{\hat{q}} = I\) and \(J_{\hat{q}} = O\)). It is easy to prove that this scheme is stable for unconstrained motion. From \(t = 7.5\) s to \(t = 15\) s the object is lifted to the initial position to make the circle. To have a better idea of how the force controller works, it is not switched on during this movement. From \(t = t_1 = 15\) s to \(t = t_2 = 55\) s the control force term is switched on, i.e., the complete control-observer scheme presented in Section III is being used only during this time period. In Fig. 4, this can be recognized because the desired force is a sinus signal. From \(t = 55\) to \(t = 62.5\) the object is put down on the table and from \(t = 62.5\) to \(t = 70\) the robots go back to their initial positions. From Fig. 4, it becomes obvious that a force control approach should be used whenever an object is being held. One can clearly appreciate when the force control term is working and when it is switched off. In the following, we discuss only the time period from \(t = 15\) s to \(t = 55\) s. The results for the tracking errors can be seen in Fig. 2 in joint coordinates, and in Fig. 3 in Cartesian coordinates. It can be appreciated that they are relatively large. This is mainly to the fact that an exact knowledge of the manipulators dynamics is required, while the models used in the experiments are not accurate. Also, only one force sensor is being used. On the other hand, for the desired force (54) the results can be considered pretty good, although one can only show the real force for the robot A465 (see Fig. 4). This demonstrates the efficacy of
designing a decentralized controller. Fig. 5 shows the observation errors. As can be appreciated, they are pretty good as well. Since the observer uses little information from the robots dynamics, this rather confirms that the tracking errors could be improved by taking into account more accurate models of the manipulators. Finally, Fig. 6 shows the input voltages. In can be observed that there are not saturation problems.

V. CONCLUSION

The tracking control problem for cooperative robots without velocity measurements is considered in this brief. The control law is a decentralized approach which takes into account motion constraints rather than the held object dynamics. By assuming that fingers dynamics are well known and that contact forces measurements are available, an observer for each finger is proposed that requires only the knowledge of the respective inertia matrix. Despite the fact that the stability analysis is complex, the controller and especially the observer are not.

Experimental results have been carried out to test the proposed approach. The test bed is made up of two industrial robots. Since only one robot owns a force sensor, the results were better for this one. However, the overall outcomes can be considered good, even though it has become clear that the approach should be modified to take into account inaccuracies in the robot model and the possible lack of force sensors.

APPENDIX I

PROOF OF LEMMA 3.1

In this Appendix, the Lemma 3.1 is proven. Recall that its main assumption is that $\mathbf{z}_i = [\mathbf{r}_i^T \Delta \mathbf{r}_i^T]_T$ is bounded and tends to zero.

a) First of all we show that $\Delta \mathbf{p}_i$ and $\Delta \mathbf{r}_i$ are bounded for all time, with $i = 1, \ldots , q^1$. To this end we use the fact that

\[ J_{\varphi_i}(\mathbf{q}_i) \mathbf{s}_i = \Delta \mathbf{p}_i + \beta_i \Delta \mathbf{r}_i - \xi_i \Delta \mathbf{F}_i \]  

or

\[ \Delta \mathbf{p}_i + \beta_i \Delta \mathbf{r}_i = J_{\varphi_i}(\mathbf{q}_i) \mathbf{s}_i + \xi_i \Delta \mathbf{F}_i. \]  

(55)

The right-hand side of (55) is bounded and tends to zero because of the assumption on $\mathbf{z}_i$. Since the left-hand side of (55) represents a stable linear filter, both $\Delta \mathbf{p}_i$ and $\Delta \mathbf{r}_i$ must be bounded and tend to zero.

b) The next step is to analyze the behavior of the tracking errors $\mathbf{q}_i$ and $\mathbf{\hat{q}}_i$. Since $\mathbf{z}_i$ is bounded, so is $\mathbf{s}_i$. According to (14), both $\mathbf{s}_p^i$ and $\mathbf{s}_d^i$ are bounded since they are orthogonal vectors. Note that $\mathbf{s}_p^i$ can be written as

\[ \mathbf{s}_p^i = Q_i(\mathbf{q}_i)(\mathbf{\hat{q}}_i + \Lambda_i \mathbf{\hat{q}}_i - \Lambda_i \mathbf{z}_i) \]

(56)

in view of the definition of $Q_i(\mathbf{q}_i)$ and $\mathbf{z}_i$. Equation (56) can be rewritten as

\[ \mathbf{\hat{q}}_i + \Lambda_i \mathbf{\hat{q}}_i = \Lambda_i \mathbf{z}_i + Q_i(\mathbf{\hat{q}}_i + \Lambda_i \mathbf{\hat{q}}_i - \Lambda_i \mathbf{z}_i) \]

bounded

\[ - J_{\varphi_i}^T \mathbf{\hat{q}}_i + J_{\varphi_i}^T \mathbf{\hat{q}}_i \]

bounded

(57)

One can be sure that the first terms in (57) are bounded because the case is being analyzed where $\mathbf{z}_i$ in (34) is bounded. The conclusion can be drawn from (8) (boundedness of $J_{\varphi_i}^T$ and $J_{\varphi_i}$), (22) and (34) (boundedness of $\mathbf{z}_i$), and (56) (boundedness of $Q_i(\mathbf{q}_i)(\mathbf{\hat{q}}_i + \Lambda_i \mathbf{\hat{q}}_i - \Lambda_i \mathbf{z}_i)$)

It is then clear that if the last term of the right-hand side is bounded, then $\mathbf{\hat{q}}_i$ and $\mathbf{\hat{q}}_i$ must be bounded since the left-hand side of (57) represents a stable linear filter. Note that

\[ J_{\varphi_i} \mathbf{\hat{q}}_i = \mathbf{p}_i - \mathbf{p}_d - (J_{\varphi_i} - J_{\varphi_i}) \mathbf{\hat{p}}_d \]

(58)

1Note that we do this for the sake of formality, since these two errors are bounded in view of the fact that both $\mathbf{p}_i$ and $\mathbf{p}_d$ are zero for the real and the desired constraints.
is bounded, with \( J_{\varphi_i} = J_{\varphi_i} (q_{i_{\text{li}}}) \). This is because revolute joints are being considered. Thus, the left-hand side of (57) can only be unbounded if

\[
J^T_{\varphi_i} J_{\varphi_i} \Lambda_i \dot{q}_i = J^T_{\varphi_i} J_{\varphi_i} \Lambda_i (q_i - q_{i_{\text{li}}})
\]

is not bounded. But in fact this term will be bounded as long as \( q_i \) is. At this point we recall that, from Assumption 2.1, \( q_i \) must satisfy the constraint \( \varphi_i (q_i) = 0 \). Furthermore, it has been assumed that none of the robots is redundant. Thus, the constraint can only be satisfied by a bounded vector \( q_i \). We can then conclude, from (57), that both \( \dot{q}_i \) and \( \ddot{q}_i \) remain bounded as long as \( x_i \) is.

To show that the tracking errors tend to zero whenever \( x_i \) does, we use the following approach. If \( x_i \equiv 0 \) then \( s_i \) becomes

\[
Q_i (q_i) \dot{q}_i + \Lambda_i \ddot{q}_i + J^T_{\varphi_i} (q_i) J_{\varphi_i} (q_i) \dot{q}_i - J_{\varphi_i} (q_{i_{\text{li}}}) q_{i_{\text{li}}} + \beta_i (q_i - \varphi_i (q_{i_{\text{li}}})) = 0. \tag{59}
\]

Suppose that \( q_i \equiv q_{i_{\text{li}}} \), then (59) becomes

\[
Q_i (q_i) \ddot{q}_i + J^T_{\varphi_i} (q_i) J_{\varphi_i} (q_i) \ddot{q}_i = 0. \tag{60}
\]

This shows that if \( \| \dot{q}_i \| \to 0 \) then \( \| \ddot{q}_i \| \to 0 \). In other words, by proving that \( \dot{q}_i \) will tend to zero one can ensure that \( \ddot{q}_i \) will do it as well. Since homogeneous constraints are being used, the following relationship is satisfied:

\[
\Delta p_i = \varphi_i (q_i) - \varphi_i (q_{i_{\text{li}}}).
\]

By developing a Taylor series around the desired trajectory \( q_{i_{\text{li}}} \) we have for \( \varphi_i (q_i) \)

\[
\varphi_i (q_i) = \varphi_i (q_{i_{\text{li}}}) + \frac{\partial \varphi_i}{\partial q_i} \bigg|_{q_i = q_{i_{\text{li}}}} (q_i - q_{i_{\text{li}}}) + \text{h. o. t.} \tag{61}
\]

There exists a positive value \( \eta_i \) small enough such that if

\[
\| \dot{q}_i \| \leq \eta_i \tag{62}
\]

then the higher order terms in (61) can be neglected and the approximation

\[
\Delta p_i = J_{\varphi_i} (q_{i_{\text{li}}}) \ddot{q}_i \tag{63}
\]

becomes valid. The following discussion will be carried out assuming that (62) holds. To prove that \( \ddot{q}_i \) tends to zero we require to find a dynamic equation which describes the behavior of this variable. It can be formed in the following fashion. From (14) one gets

\[
\ddot{q}_i^T s_{pi} = \ddot{q}_i^T \dot{q}_i + \ddot{q}_i^T \Lambda_i \dot{q}_i - \ddot{q}_i^T \Lambda_i z_i
\]

with \( P_i = J^T_{\varphi_i} (q_i) J_{\varphi_i} (q_i) \). In view of the fact that

\[
\ddot{q}_i^T \dot{q}_i = \| \dot{q}_i \| \frac{d}{dt} \| \dot{q}_i \|,
\]

one can rewrite (64) as

\[
\frac{d}{dt} \| \dot{q}_i \| = -\ddot{q}_i^T \Lambda_i \dot{q}_i + \ddot{q}_i^T P_i \dot{q}_i + \ddot{q}_i^T P_i \Lambda_i \dot{q}_i - \ddot{q}_i^T \Lambda_i z_i + \ddot{q}_i^T s_{pi} \tag{65}
\]

To develop the right-hand side of this equation we analyze the term \( \ddot{q}_i^T P_i \dot{q}_i \) first. From (58)

\[
\Delta p_i = J_{\varphi_i} \dot{q}_i + (J_{\varphi_i} - J_{\varphi_i}) q_{i_{\text{li}}}.
\]

By multiplying (63) by \( \beta_i \) and adding the result to (66) one gets

\[
J_{\varphi_i} \ddot{q}_i + \beta_i J_{\varphi_i} \dot{q}_i = \Delta p_i - (J_{\varphi_i} - J_{\varphi_i}) q_{i_{\text{li}}} + \beta_i \Delta p_i.
\]

Thus we have

\[
\ddot{q}_i^T P_i \dot{q}_i = \ddot{q}_i^T J^T_{\varphi_i} (J_{\varphi_i} J_{\varphi_i})^{-1} J_{\varphi_i} \dot{q}_i
\]

\[
= -\beta_i \ddot{q}_i^T P_i \ddot{q}_i + \beta_i \ddot{q}_i^T J^T_{\varphi_i} \Delta p_i + \ddot{q}_i^T J^T_{\varphi_i} \Delta p_i
\]

\[
- \ddot{q}_i^T \ddot{q}_i (J_{\varphi_i} - J_{\varphi_i}) (q_{i_{\text{li}}} - \beta_i \dot{q}_i).
\]

Substituting in (65) one gets

\[
\| \dot{q}_i \| \frac{d}{dt} \| \dot{q}_i \| = -k_i \| \ddot{q}_i \|^2 + (k_i - \beta_i) \ddot{q}_i^T P_i \dot{q}_i
\]

\[
- \ddot{q}_i^T J^T_{\varphi_i} (J_{\varphi_i} - J_{\varphi_i}) (q_{i_{\text{li}}} - \beta_i \dot{q}_i)
\]

\[
+ \ddot{q}_i^T J^T_{\varphi_i} (\Delta p_i + \beta_i \Delta p_i)
\]

\[
+ k_i \ddot{q}_i^T Q_i z_i + \ddot{q}_i^T s_{pi} \tag{67}
\]

where the definition \( \Lambda_i = k_i I \) has been used. In view of Assumption 2.2, the following bound can be established:

\[
\| \dot{q}_i \| \frac{d}{dt} \| \dot{q}_i \| \leq \gamma_i \| \dot{q}_i \| \tag{68}
\]

with

\[
\gamma_i \triangleq \alpha_i^2 C_{\text{max}} L_i (v_{\text{mi}} + \beta_i \eta_i). \tag{69}
\]

\( v_{\text{mi}} \) is the bound for the velocity vector \( \dot{q}_i \), i.e., \( \| \dot{q}_{i_{\text{li}}} \| \leq v_{\text{mi}} \) for all time. On the other hand, from (14) you have

\[
s_{pi} = s_i - J^T_{\varphi_i} (q_i) (\Delta p_i + \beta_i \Delta p_i - \xi_i \Delta F_i). \tag{70}
\]

By substituting (68)–(70) in (67), taking norms and dividing by \( \| \dot{q}_i \| \) one gets

\[
\frac{d}{dt} \| \dot{q}_i \| \leq -\alpha_i \| \dot{q}_i \| + \| s_i \|
\]

\[
+ \lambda_{\text{max}} (\xi_i) C_{\text{max}} \| \Delta F_i \| + k_i \| z_i \| \tag{71}
\]

with

\[
\alpha_i \triangleq k_i - |k_i - \beta_i| - \gamma_i. \tag{72}
\]

Note that \( k_i, \beta_i, v_{\text{mi}}, \) and \( \eta_i \) can always be chosen so as to get \( \alpha_i > 0 \) and that the last three elements of the right-hand side of (71) are bounded since \( x_i \) is bounded. However, (71) is valid only if (62) holds. Thus, in the end, it is not enough for \( x_i \) to be bounded. We must find a bound \( x_{\text{max}} \), such that (62) is valid. In order to guarantee this, \( x_{\text{max}} \) should appear in (71) explicitly. Of course, if

\[
\| z_i \| \leq \| \ddot{q}_i \| \leq \| x_i \| \leq x_{\text{max}},
\]

then one has

\[
\| x_i \| = \| s_i \| + \| s_i \| \leq \| \dot{q}_i \| \leq x_{\text{max}} \tag{73}
\]

with \( r_i = z_i + \Lambda_i z_i \). Since in (71) one has \( z_i \) and not \( r_i \), it should be noted that every element of \( r_i \) is given by

\[
r_{ij} = \dot{z}_i + k_i z_{ij}, \quad \text{for } j = 1, \ldots, n_i.
\]

This is a stable linear filter with gain \((1/k_i), i.e., one has \( |z_{ij}| \leq \frac{1}{k_i} \).
tends to zero, it is clear from (71) that \( \| \tilde{z}_i \| \leq \sqrt{\frac{n_i}{k_i}} x_{\text{max}} \) \( \forall i \). Thus it is straightforward to show that

\[
\| z_i \| \leq \frac{\sqrt{n_i}}{k_i} x_{\text{max}} \quad (74)
\]

which allows to rewrite (71) as

\[
\frac{d}{dt} \| \hat{q}_i \| \leq -\alpha_i \| \hat{q}_i \| + \sigma_i
\]

with

\[
\sigma_i \triangleq (1 + \lambda_{\text{max}}(\xi_i)) C_{0i}^T + \sqrt{n_i} x_{\text{max}} \cdot \quad (75)
\]

From the Comparison Lemma [13], it is

\[
\| \hat{q}_i(t) \| \leq \frac{\sigma_i}{\alpha_i} + \exp(\frac{-\alpha_i}{\sigma_i})(\| \hat{q}_i(0) \| - \frac{\sigma_i}{\alpha_i}) \quad (76)
\]

for all time. Furthermore, (62) will be satisfied if

\[
\| \hat{q}_i(0) \| \leq \frac{\sigma_i}{\alpha_i} = \left( 1 + \lambda_{\text{max}}(\xi_i) C_{0i}^T + \sqrt{n_i} x_{\text{max}} \right) \frac{\eta_i}{\alpha_i} \leq \eta_i. \quad (77)
\]

Equation (78) can be easily accomplished if

\[
x_{\text{max}} \leq \frac{\eta_i \alpha_i}{1 + \lambda_{\text{max}}(\xi_i) C_{0i}^T + \sqrt{n_i}}. \quad (79)
\]

Thus, if \( \| z_i \| \) tends to zero, it is clear from (71) that \( \| \hat{q}_i \| \) will tend to zero as well. Recall that this implies the convergence to zero of \( \| \hat{q}_i \| \).

c) When \( z_i \) and thus \( \Delta \lambda_i \) tends to zero, \( \Delta \lambda_i \) does not necessarily do it nor remain bounded. In order to prove this, one may use the fact that \( J_{\varphi_i}(q_i)s_i = \Delta p_i + \beta_i \Delta p_i - \xi_i \Delta F_i \). The last equality is valid if constraint (6) holds. Thus you have

\[
J_{\varphi_i}(q_i)s_i = -J_{\varphi_i}(q_i)s_i - \xi_i \Delta \lambda_i. \quad (80)
\]

To get rid of \( s_i \), we have from \( (6), (28) \)

\[
H_i(q_i)s_i - J_{\varphi_i}^T(q_i) \Delta \lambda_i = h_i \quad (81)
\]

where

\[
h_i \triangleq -C_i(q_i, \dot{q}_i)s_i - K_{\text{DR}i}s_i + K_{\text{R}i}r_i + k_{F'i}J_{\varphi_i}^T(q_i) \Delta F_i - C_i(q_i, \dot{q}_i)s_i + H_i(q_i)\dot{e}_i(r_i). \]

It is worth pointing out that \( h_i(t) \) is bounded since \( \dot{q}_i \) is assumed to be bounded. Combining (80) and (81) one gets

\[
\Delta \lambda_i = -(J_{\varphi_i}(q_i)H_i^{-1}(q_i)J_{\varphi_i}(q_i))^{-1} \cdot (J_{\varphi_i}(q_i)s_i + J_{\varphi_i}(q_i)H_i^{-1}(q_i)h_i(t)). \quad (82)
\]

Since the inverse exits always because \( J_{\varphi_i}(q_i) \) is full rank we conclude that \( \Delta \lambda_i \) must be bounded because the right-hand side of (82) is bounded. Note that from (81), \( s_i \) remains bounded too. Finally, if \( x_i \) tends to zero, so do \( s_i \) and \( h_i(t) \) and we arrive to the conclusion that \( \Delta \lambda_i \) tends to zero as well.

**Appendix II**

**Proof of Theorem 3.1**

Theorem 3.1 can be proven as follows. Consider the Lyapunov candidate function (36), where \( s_i(0) \) belongs to the region (39). If \( \dot{V}_i(x_i) \leq 0 \) with \( \dot{V}_i(x_i) = 0 \) if and only if \( x_i = 0 \), then the maximum value that the norm of \( x_i \) may have is given by (49). According to the proof of Lemma 3.1, \( x_{\text{max}} \) must satisfy (79) so that \( \hat{q}_i \) and \( \hat{q}_i \) may tend to zero if \( x_i \) does. In view of (49), condition (79) can be satisfied if (40) holds.

Of course, it remains to show that \( \dot{V}_i(x_i) \leq 0 \) actually happens. Rewrite (36) as

\[
V_i = \frac{1}{2} s_i^T H_i s_i + \frac{1}{2} r_i^T H_i r_i + \frac{1}{2} \Delta F_i^T \xi_i \Delta F_i. \quad (83)
\]

The derivative of (83) along (15), (28), and (33) can be computed as

\[
\dot{V}_i(x_i) = \frac{1}{2} s_i^T H_i(q_i)s_i + \frac{1}{2} r_i^T H_i(q_i)r_i + \Delta F_i^T \xi_i \Delta \lambda_i + k_{F'i} \Delta F_i \xi_i \Delta F_i. \quad (84)
\]

To simplify (84) one should take into account that \( s_i^T K_{\text{DR}i}r_i - r_i^T K_{\text{DR}i}s_i = -s_i^T D_{\text{DR}i}s_i \). Also, note from (16) that

\[
s_i^T J_{\varphi_i}(q_i) (\Delta \lambda_i + k_{\text{F'i}} \Delta F_i) = -\Delta F_i^T \xi_i \Delta \lambda_i - k_{\text{F'i}} \Delta F_i \xi_i \Delta F_i. \]

The last equality is valid in view of Property 2.5 and by the fact that constraint (6) must be satisfied for \( p_i, \dot{p}_i, p_{ib}, \) and \( \dot{p}_{ib} \). Thus, by taking Property 2.2 into account, (84) becomes

\[
\dot{V}_i(x_i) = -k_{F'i} \Delta F_i^T \xi_i \Delta F_i - s_i^T K_{\text{DR}i}s_i - r_i^T H_i(r_i)s_i + s_i^T C_i(q_i, \dot{q}_i)s_i + \dot{s}_i^T H_i(q_i)e_i(r_i). \quad (85)
\]

Since \( x_i(0) \) is in region (39), you must have that

\[
\dot{V}_i(x_i(0)) \leq -k_{F'i} \lambda_{\text{min}}(\xi_i) \| \Delta F_i \|^2 - (\lambda_{\text{min}}(K_{\text{R}i}) - \mu_{2i}) \| r_i \|^2 + (\mu_{3i} + \kappa_{2i}) \| s_i \|, \quad (86)
\]

By taking conditions (42)-(43) into account one gets

\[
\dot{V}_i(x_i(0)) \leq -\delta_1 \| s_i \|^2 - \delta_2 \| r_i \|^2 - k_{F'i} \lambda_{\text{min}}(\xi_i) \| \Delta F_i \|^2. \quad (87)
\]

Thus, \( \dot{V}_i(x_i(0)) \) is negative. But for continuity of the solution, \( \| x_i \| \) cannot become larger than \( x_{\text{max}} \) as long as \( \dot{V}_i(t) \) is negative. Since in fact this is the only condition we needed to get (87), one concludes that \( \dot{V}_i(x_i) \leq 0 \) \( \forall t \geq 0 \), and that \( \dot{V}_i(x_i) = 0 \) if and only if \( x_i = 0 \). Thus \( x_i \) tends to zero and is bounded by \( x_{\text{max}} \).

Now, from definition (22) one has directly

\[
\lim_{t \to \infty} z_i = 0 \quad \lim_{t \to \infty} \dot{z}_i = 0. \]
From Lemma 3.1 a) and b), we get
\[
\lim_{t \to \infty} \Delta \dot{p}_i = 0 \quad \lim_{t \to \infty} \Delta p_i = 0 \quad \lim_{t \to \infty} \dot{\lambda}_i = 0 \quad \lim_{t \to \infty} \ddot{\lambda}_i = 0.
\]
To applied c) of Lemma 3.1, we only need to show that \( \dot{\lambda}_i \) is bounded. This is certainly the case because \( \dot{\lambda}_i \) and \( \ddot{\lambda}_i \) are bounded. Thus we get
\[
\lim_{t \to \infty} \Delta \lambda_i = 0.
\]
Finally, the stability of the whole system can be proven using
\[
V = \sum_{i=1}^{I} V_i(x_i).
\]

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