Non-Rigid Registration and Geometric Approach for Tracking in Neurosurgery

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Abstract

Registration of points obtained from preoperative 3D model and points of intraoperative stereo 3D representation, as well as tracking of surgical devices are two important stages in computer aided surgery (CAS). This work shows the application and comparison of two different methods (RPM and ICP) in the registration process of patient’s head markers, as well as the use of geometric methods to track the surgical device in real time.

1. Introduction

Around the world there is great interest in developing systems to help doctors and surgeons. Such systems are useful planning and/or executing surgeries because they help to realize minimally invasive surgeries and improve the accuracy of them. In such systems, as well as in other areas of computer vision, we find the registration problem.

We can think in registration as the way to relate 3D points of the image on the computer to points in the patient in the operating room, so registration problem is to find the transformation between two points sets. If we don’t know the correspondences between these points, we have to estimate their correspondences and the transformation, becoming the problem more difficult. In this work we compare the results obtained in applying one version of the popular ICP algorithm and the TPS-RPM algorithm (proposed by Haili Chui and Anand Rangarajan in [1]) to register the points set selected from computer tomographic (CT) images and its counterpart obtained from triangulating points of the patient’s head observed by a stereo camera system. We show how the RPM method gives better results than ICP aligning more precisely the two points sets. Once the registration process is complete, the next step is to track the surgical device model. We define the line which describes the surgical device and use the geometric algebra methods to estimate its motion. Structure of the paper is as follows: the second section gives a brief introduction to the concepts of geometric algebra used to estimate the motion of the surgical device; section three gives a brief description of the ICP and RPM methods used in registration process. The fourth section shows the results obtained applying the methods for registration and tracking. Finally, section five contains conclusions and observations.

2. Geometric algebra

Geometric algebra is a coordinate-free approach to geometry based on the algebras of Grassmann and Clifford. The algebra is defined on a space whose elements are called multivectors; a multivector is a linear combination of objects of different type, e.g. scalars and vectors. It has an associative and fully invertible product called the geometric or Clifford product. The existence of such a product and the calculus associated with the geometric algebra endows the system with tremendous power. Geometric algebra provides a very natural language for projective and affine geometries and it has all the necessary equipment for the tasks involving the meet and join operations as well as directed distance. Further information can be found in [4], here we only present the concepts which will be used in tracking stage.

2.1. Affine plane and horosphere

The n-dimensional affine plane $A_n (\mathbb{R}^{p,q})$ is an homogeneous representation of points in the $\mathbb{R}^{p,q}$ space which extends the space $\mathbb{R}^{p,q}$ to the projective space $\mathbb{R}^{p,q+1}$ by use of a null vector:

$$x_h = x + \eta \in A_n (\mathbb{R}^{p,q})$$

(1)

The $(p,q)$—horosphere is usually defined as:

$$H^p_q (\mathbb{R}^{p,q}) = \left\{ \frac{1}{2} x_h \eta_0 x_h | x_h \in A_n (\mathbb{R}^{p,q}) \right\} \in \mathbb{R}^{p+1,q+1}$$

(2)

In general, a k-plane $A^k$ consist of a moment of grade k and a direction of grade $k+1$. The distance between an $(r-1)$-plane $A^h = a_0^h A \ldots A a_s^h$ and a $(s-1)$-plane $B^h =$
affine plane, the wedge product between they defines an

tate at the left by R and a! the right by its conjugate R. Sim-

malarly, translations are made by means of the

recented one of this algorithms can be found in [1] and it uses two tech-

niques to solve the correspondence problem:

- Soft assign: the basic idea is allow the matrix of corre-

spondences M to take continuous values in the inter-

val [0, 1]; this “fuzzy” correspondences improve gradually dur-

ng optimization without jump in the space of permuta-

ions of binary matrices.

- Deterministic annealing: technique used to control the

fuzzy of the correspondences by means of an en-

ropy term in the cost function (called energy func-

ion) introducing a parameter T of temperature which

is reduced in each stage of the optimization pro-

cess beginin at a value T0 and reducing by a factor

until reach some final temperature.

Using these two techniques, the problem is to minimize the

function:

\[
E(M, f) = \sum_{i=1}^{n} \sum_{a=1}^{k} m_{ai} ||x_i - f(v_a)||^2 + \lambda ||Lf||^2 + T \sum_{i=1}^{n} \sum_{a=1}^{k} m_{ai} \log m_{ai} - \zeta \sum_{i=1}^{n} \sum_{a=1}^{k} m_{ai}
\]

for that, we will follow the next two steps:

**Update the correspondence:** for points \( v_a, a = 1, 2, ..., k \)

\( x_i, i = 1, 2, ..., n \) modify \( m_{ai} \):

\[
m_{ai} = \frac{1}{T} e^{-\frac{||x_i - f(v_a)||^2}{T}}
\]

for outliers entries \( a = k + \tau \) and \( i = 1, 2, ..., n \):

\[
m_{k+1,i} = \frac{1}{T_0} e^{-\frac{||x_i - f(v_{k+1})||^2}{T_0}}
\]

and for outliers entries \( a = 1, 2, ..., k \) and \( i = n + \tau \):

\[
m_{a,n+1} = \frac{1}{T_0} e^{-\frac{||x_{n+1} - f(v_a)||^2}{T_0}}
\]

**Update the transformation:** here we need to solve the fol-

owing least-squares problem:

\[
\min \ E(f) = \min \ \sum_{i=1}^{n} \sum_{a=1}^{k} m_{ai} ||x_i - f(v_a)||^2 + \lambda T ||Lf||^2
\]
But Chui H. et al proposed [1] the use of thinplates splines [2] and, by means a QR-decomposition, the final function is:

\[
E_{ps}(d, w) = \|Y - Vd - \Phi w\|^2 + \lambda_1(w^T\Phi w) + \lambda_2[d - I]^T(d - I)
\]

(12)

### 3.2. ICP

The Iterated Closest Point (ICP) algorithm is quite similar to the RPM algorithm. The main difference is that ICP depends on the nearest-neighbor heuristic and don’t guarantee the correspondence be one-to-one. It also follows the two step process.

**Update the correspondence:** to obtain the correspondences, we compute the distance between each pair of points \((v_i, x_j); i = 1, \ldots, k; j = 1, \ldots, n\). Then we search the smallest distance from each point \(v_i\) to \(x_j\) and compare it with a preestablished threshold; if distance is smaller than it, this two points could be corresponding points, so \(m_{i,j} = 1\), otherwise \(m_{i,j} = 0\).

**Update the transformation:** here we follow the same strategy than in RPM algorithm because we have the matrix of correspondences and compute the transformation with (12).

### 4. Tracking of surgical device

Actually there exist some algorithms to track objects in 2D images; one of them is the well known Shi-Tomasi-Kanade tracker. The reader could see the details of this algorithm in [3]. We prefer explain how we compute the transformation between lines defining the surgical device. For clarity, consider figure 3. The first step is to form the line defining the surgical device (SD); this is done by selecting two points on the SD in the stereo images, then triangulating this points and applying the wedge product between the resulting 3D points: \(L_1 = X_1 \wedge X_2\). The next step is to move the line \(L_1\) in stereo camera coordinates to the real position \(L_2\). Here we apply the concepts of geometric algebra as follows. First, we obtain the distance between the two lines using (3)

\[
d = \frac{\eta_0 \cdot (L_1 \wedge L_2)}{(\eta_0 \cdot L_1) \wedge (\eta_0 \cdot L_2)}
\]

(13)

then we form the plane containing line \(L_1\) and the vector representing the distance \(P = L_1 \wedge d\) and then apply the meet operation to find the point on line \(L_2\) closest to \(L_1\):

\[
p_2 = \text{meet}(P, L_2) = (P \cdot (L_1 \wedge \eta_0)) \cdot L_2
\]

(14)

Now we form the translator \(T_{orig}\) moving \(L_1\) to the origin by means of (15) and the translator moving \(L_1\) to \(L_2\) by means of (16):

\[
T_{orig} = \exp\left(-\frac{\eta_0 p_1}{2}\right) = 1 - \eta_0 p_1 = 1 - \eta_0 (d - p_2)
\]

(15)

\[
T_{L_1L_2} = \exp\left(\frac{\eta_0 d}{2}\right) = 1 + \eta_0 d
\]

(16)

To find the rotation, we compute the angle \(\theta\) by means of (17); the rotation axis \(w = (\eta_0 \cdot L_1) \times (\eta_0 \cdot L_2)\) and finally form the rotor as given by (18).

\[
\theta = \arccos\left(\frac{\eta_0 \cdot L_1 \cdot \eta_0 \cdot L_2}{\|\eta_0 \cdot L_1\| \|\eta_0 \cdot L_2\|}\right)
\]

(17)

\[
R_{L_1L_2} = \exp\left(\frac{-\theta}{2} w^*\right)
\]

(18)

The total transformation of \(L_1\) to \(L_2\) is given by (19) where \(M = T_{L_1L_2}R_{L_1L_1}\)

\[
L_2 = T_{orig} \left(M_{L_1L_2} T_{orig} L_1 M_{L_1L_2} T_{orig}\right) T_{orig}
\]

(19)

### 5. Experimental results

We use for the test a human skull. All the algorithms were implemented in C++ under Linux. Figure 1 shows the errors obtained by ICP and RPM methods which is measured as the magnitude of the distance between the resulting \(p_r\) and the expected \(p_0\) point coordinates \(e_i = \sqrt{(x_{r_i} - x_i)^2 + (y_{r_i} - y_i)^2 + (z_{r_i} - z_i)^2}\); the reader could see the RPM method gives much better results than ICP (actually, the minimum error obtained with RPM was 45.5 voxels while with ICP was 11.1 voxels). Figure 2 shows the alignment of points in the human skull. The initial points set was formed by selecting points on the skull defined by markers placed on it and triangulating them according to stereo images; the final points set was formed with the same points but selected from 3D virtual model constructed with the CT images. Figure 3 shows the position of SD according to stereo camera system (fig. 3a) and the expected position (fig 3b). Figures 4 and 5 shows the results of tracking the SD in real time in 2D images and in 3D representation.

### 6. Conclusions

We have tested two algorithms for the alignment of points in presence of non-rigid transformations and then proved the results in the registration of points of a human skull according to 3D model and stereo camera system. We have underlined the superiority of RPM method in this process in comparison with ICP method. Also we have implemented the tracking of SD for real time combining techniques for
Figure 1. Errors in aligning the points sets using ICP and RPM methods.

Figure 2. Result of registration of points on human skull using a) RPM method and b) ICP method (see at the top a magnification).

Figure 3. Calibrating the line position of SD: a) position according stereo system; b) expected real position.

Figure 4. Left and right images from stereo system.

Figure 5. Result of tracking: first it is estimated the motor and then applied to each new position of SD in a 3D virtual world.

objects tracking in 2D images and geometric methods to find the rigid transformation relating two lines (which is done by means of motors in the 3D space.

References


