A TS Fuzzy Observer for an Anaerobic Fluidized Bed

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Abstract—In this article, a Takagi-Sugeno fuzzy observer for the anaerobic digestion process in a fluidized bed reactor is proposed. First, a general explanation of this bioprocess is introduced. A model of the system discretized in the space by the orthogonal collocation method is presented. This model is analyzed locally to prove properties of stability and observability. Then, the design of the fuzzy observer is presented; the choice of the local models was made by the principal component analysis. The observer performances are shown via realistic simulations.

I. INTRODUCTION

Bioprocesses are among the more difficult systems to model. The presence of biological organisms induces generally nonlinear expressions in the mathematical models. In the case of the anaerobic digestion process in a fluidized bed reactor (FBR), the hydrodynamic behaviour may be also a source of problems to model and to control. The complexity of the processes and the model structures involve some difficulties to design control strategies. But, before the processes control, there exists an additional problem: the presence of non measurable variables, such as the micro organisms; then, in the real implementations, a careful choice of the sensors is recommended to develop observers which allow estimating the non measurable variables. The observability and estimation problem of bioprocesses has been studied by some authors as shown in [1] – [5]. In this paper, an anaerobic digestion process application is considered: paper industry effluents treatment. A fuzzy observer for an anaerobic fluidized bed reactor is proposed using the Takagi–Sugeno approach ([6], [10]). The goal of the observer is to estimate the state variables of the process, specially the biomass, from measurements of biogas flow rate. The idea is to linearize the model around different operating points selected from a principal component analysis (PCA), a statistical tool used for the behaviour analysis of bioprocesses ([11] – [15]); after that, a classical observer is designed for each local model and then, the global state is estimated from a fuzzy interpolation.

In next sections the biological, physico-chemical and hydrodynamic phenomena are considered to deduce a model of anaerobic digestion in FBR. Next, a brief analysis of the mathematical model is shown. Subsequently, a fuzzy observer is developed and validated via simulations and finally some conclusions and perspectives are stated.

II. THE ANAEROBIC DIGESTION PROCESS IN A FLUIDIZED BED REACTOR

The anaerobic digestion is an efficient process to treat effluents with a high organic load, such as the paper industry ones. The process is composed by four successive stages: hydrolysis, acidogenesis, acetogenesis and methanogenesis. The input complex molecules (substrate) are degraded by means of anaerobic bacteria (biomass) producing biogas: methane (CH₄) and carbon dioxide (CO₂). From an automatic control point of view, modelling anaerobic digestion is a hard work due to the different stages dynamics and process phenomena.

A. Biological and physico-chemical phenomena

It is possible to differentiate between very fast stages (acidogenesis and acetogenesis), fast stage (hydrolysis) and slow stage (methanogenesis). The last one is considered as the limiting stage because it is the slowest one and it is very sensitive to inhibitions by an excess of acids, as shown in [16] – [18]. Dynamic of the very fast stages is assumed to be neglected. Hydrolysis dynamic cannot be neglected because it is faster than methanogenesis but not as acidogenesis and acetogenesis. Then it is possible to consider the two phases of figure (1). Slow stage corresponds to methanogenesis and fast stages are the others phases of the process.

Fig. 1. Functional diagram of the anaerobic digestion

In this paper, only the methanogenesis is considered because it is the limiting stage. Then, the model proposed for this one is composed by a set of five algebraic equations
and four ordinary differential equations. From the acid-base equilibrium between acetate (S) and non ionized acetic acid (HS) two equations are deduced; first one, for the substrate conservation, and second one, for acid-base equilibrium with equilibrium constant $K_a$. An algebraic equation is stated considering inorganic carbon production (IC) from bicarbonate (B) and carbon dioxide $CO_{2D}$ and another equation for equilibrium between them by means of $K_b$ constant. The fifth algebraic equation represents electroneutrality in the reactor, where $Z$ stands for the cations. The set of algebraic equations is presented below:

$$
\begin{align*}
H^+ S^- - K_a HS &= 0 \\
HS + S^- - S_2 &= 0 \\
H^+ B - K_a CO_{2a} &= 0 \\
B + CO_{2a} - IC &= 0 \\
B + S^- - Z &= 0 \\
\end{align*}
$$

Ordinary differential equations are used to represent biomass evolution and substrate consumption. The substrate produced in the fast stages is used as the input substrate for the methanogenesis. This substrate is considered as acetic acid equivalent and it is named $S_2$. The bacteria population (biomass) is known as $X_2$. The biomass grows at a rate $\mu$ consuming the substrate. $Y_1$ is the consumption yield coefficient. Inorganic carbon evolution is a result of the biological phase and the law of partial pressure for the dissolved carbon dioxide must be considered. In addition, cations are biologically inert. Then, dynamic part of the model is presented now:

$$
\begin{align*}
\frac{dX}{dt} &= \mu X_2 \\
\frac{dS_2}{dt} &= -Y_1 \mu X_2 \\
\frac{dIC}{dt} &= \lambda Y_1 \mu X_2 \\
\frac{dZ}{dt} &= 0 \\
\end{align*}
$$

where $\mu = \frac{\mu_{max} HS}{k_s + HS + \frac{HS^2}{K_a}}$ and $\lambda = \frac{CO_{2D}}{P K_a + CO_{2D}}$

Finally, methane and carbon dioxide flow rates are stated as process outputs. Both variables are typically measured in this kind of process.

$$
\begin{align*}
Q_{CH_4} &= Y_1 Y_2 \mu X_2 \\
Q_{CO_2} &= \lambda Y_1 Y_2 \mu X_2 \\
\end{align*}
$$

B. Hydrodynamic phenomena

Hydrodynamic behaviour of a FBR (figure 2) is complex due to spatial distribution, and the substrate feedback.

Fluidization is an operation to keep homogeneous agitation of solid particles in a liquid or gaseous environment. In this case, hydrodynamic of soluble components was deduced from experiments and result was a piston with axial dispersion behaviour as in [19].

$$
\frac{\partial x(t,z)}{\partial t} = -U_1 \frac{\partial x(t,z)}{\partial z} + D_a \frac{\partial^2 x(t,z)}{\partial z^2} 
$$

where $U_1$ is interstitial speed and $D_a$ is the axial dispersion coefficient.

This is a distributed parameters system with an infinite dimension. To approach the system to a finite dimension model there exist several methods, for example: pondering remaining and orthogonal collocation; first one is an exact solution but the generated model is complex with a high dimension, hard to use in a control perspective; second one is not an exact solution but the model is easier to analyze and manipulate in a control perspective, in addition, the physical sense is respected, as explained in [20]. Thus, second method was chosen. A spatial discretization in four points (four different heights in the reactor) was considered. Now, the model depends on time only.

C. The complete process

Axial dispersion affects soluble components such as substrate, inorganic carbon and cations. Since biomass is attached to a solid material (biolite), it follows a completely stirred behaviour. The bacteria attached to biolite are considered as active biomass and they are represented by the growth rate ($\mu$). The other bacteria are considered as inactive biomass and they are represented by the death rate ($k_d$). In general, $k_d$ is assumed to follow a first order kinetics; so, it can be deduced from substrate consumption measures.

In the complete model, algebraic part is composed by twenty equations since (1-5) are valid in every collocation point. Differential equations are modified as follows:
\[
\frac{dx_i}{dt} = (\mu - k_i)x_i \\
\frac{ds}{dt} = \mu x_i + \left[ \lambda + \sum_{j=1}^{r} i_j \right] s_i + \sum_{j=1}^{r} \left( (1 - r) s_i + r s_i^* \right) \\
\frac{dc}{dt} = \lambda y \mu x_i + \left[ \lambda + \sum_{j=1}^{r} i_j \right] c_i + \sum_{j=1}^{r} \left( (1 - r) c_i + r c_i^* \right) \\
\frac{dz_i}{dt} = \lambda y \mu x_i + \left[ \lambda + \sum_{j=1}^{r} i_j \right] z_i + \sum_{j=1}^{r} \left( (1 - r) z_i + r z_i^* \right)
\]

where \( \lambda_y = \frac{K_{u} - 2CO_{2d}}{K_{u} - CO_{2d}} \), \( \mu = \frac{\mu_{max} HS}{k_y + HS + HS^2 / k_y} \), \( r \) is the feedback flow rate, \( i \) is the collocation matrix, \( i = 1, \ldots, 4 \) is the index of collocation points and the upper index \( tr \) indicates a function depending on the time and the feedback flow (delay).

D. Model Analysis

Due to the model complexity associated with non-linearity, the analysis of stability, observability and controllability is made locally around an experimental operating point. Different theoretical operating points were considered to have results more generals. The system is locally stable, observable and controllable around the experimental operating point determined in [21]. The theoretical operating points are chosen considering different values in a valid interval of the input variables. In fact, the input variations lead the system from an initial equilibrium point to new equilibrium points; the non-linear system was linearized and analyzed around the new equilibrium point. The input variables and its respective considered values are shown in Table (1).

Three hundred input scenarios are possible from these intervals. In [22] a classic stability analysis for the linearized models is presented. They are stable for the input intervals considered; there exists an instability problem only for \( \text{Qin}>12 \) l/hr. The observability analysis was made using the Rosenbrock theorem considering the biga measure. The linear systems issued from an input scenario with small \( \text{Qin} \) and \( \text{S2in} \) variations do not verify the observability condition.

III. FUZZY OBSERVER DESIGN

The principal objective of this fuzzy observer is to estimate the state variables of the anaerobic digestion process, specially the biomass evolution because this is an important immeasurable variable, which can be used to design control strategies. Different observer approaches were tested, such as the asymptotic, robust and sliding horizon ones ([23] – [25]). The principal problem of these observers is the numerical instability and the calculus time. In order to avoid these problems a Takagi-Sugeno approach ([26] – [27]) is considered. In this approach, the non-linear model is linearized around different operating points, a classic observer for each local model is designed and then the global state is calculated by an interpolation of the local estimated states. To choose the local models, a principal component analysis was developed.

A. Takagi-Sugeno Approach

A Takagi-Sugeno system is represented by \( r \) if then fuzzy rules, the premises are composed by linguistic terms and the consequents are functions, in this case, a state space system. The \( i^{th} \) rule is defined as follows:

IF \( z_i(t) \) is \( F_{ij} \) and \( \ldots \) and \( z_g(t) \) is \( F_{ig} \)

THEN \( \hat{x}(t) = A_x(t) + B_{u(t)} \)

\( y_i(t) = C(t)x_i(t) \)

\( \eta \) is known as the membership function and it is calculated as follows:

\[
\eta = \frac{\prod_i F_{ij}[z_i(t)]}{\sum_i \prod_j F_{ij}[z_i(t)]}
\]

where \( [z(t)] = [z_1(t) \ldots z_g(t)] \), \( F_{ij}[z(t)] \) is the membership degree of \( z_i(t) \) in \( F_{ij} \) and for all \( i: \eta_i \geq 0 \) and \( \sum_i \eta_i = 1 \).

If the observability condition is verified, a classic observer for each local model can be designed; if not, the observable and non-observable states must be separated to design an observer for the observable part. The global estimated state results from the interpolation of the local active observers using the Takagi-Sugeno approach. The \( i^{th} \) rule for the observer is:

IF \( z_i(t) \) is \( F_{ij} \) and \( \ldots \) and \( z_g(t) \) is \( F_{ig} \)

THEN \( \dot{x}(t) = A_x(t) + B_{u(t)} + K_i (y(t) - \hat{y}(t)) \)

\( \hat{y}(t) = C(t)x_i(t) \)

where \( K_i (i = 1, \ldots, r) \) represent the observer matrix gain.

And the global estimated state is:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i A_x(t) + \sum_{i=1}^{r} \mu_i B_{u(t)} + \sum_{i=1}^{r} \mu_i [y(t) - \hat{y}(t)]
\]

\[
\hat{y}(t) = \sum_{i=1}^{r} \mu_i C(t)x_i(t)
\]
B. Principal Component Analysis

A careful analysis of the inputs variation intervals is required to choose the set of the operating points, and consequently the set of local models, which allows having the best representation of the process ([28] – [29]). The different operating points generated by the three hundred input scenarios were analysed by means of the PCA. This method is used to simplify in the best way a big data set composed by different behaviours (variables) of each member (individus) in a studied population. The idea is to determine the principal components, which characterize in the best way the whole of the analyzed information. The data interpretation from this grouping is easier than from the whole data set. In the considered case, the PCA shows that two principal components represent 80% of the whole data set.

After that, the analysis concerns the individus, in this case, the three hundred local operating points. The figure (3) shows the dispersion diagram.

To ease the diagram interpretation, only two hundred models are included in figure (3). The dispersion diagram illustrate that local models are grouped following S2in (vertically) and Qin (horizontally) values. This is an indicator of the relevance of the two variables, which are chosen as fuzzy variables to select local models. When the three hundred models are considered, vertical and horizontal grouping become hard to determine. It means that likely there exist another input variable to group the local models. Then, the objective of next analysis is to find existing relations between the variables (input scenarios, poles of the linear systems and methane production) to clarify how models are grouping. Figure (4) shows the correlations circle.

Variables are classified in six groups. First one, composed by corresponding poles to IC_{1-4} and S_{34}, is anticorrelated with group 2 (Q_{in}). That is meaning Q_{in} affects group 1. Since group 2 (composed by corresponding poles to S_{21-3}) is near to group 1, it could be influenced too by Q_{in}. This confirms the Q_{in} relevance on process behaviour. There exist an anticorrelation between groups 4 and 2, but neither involves input variables. Fifth group contains the input variables S_{2in}, IC_{in} and Z_{in}, but it is not correlated to other groups. Finally, group 6 shows the strong correlation between corresponding pole to X_{2} and pH since they are very near. That is meaning pH must be considered as a variable to select local model because it affects directly X_{2}, which imposes the global process dynamic. Then Q_{in}, S_{2in} and pH are taken as fuzzy variables.

C. Fuzzy observer implementation

From the previous analysis, three fuzzy variables and forty-five local models were chosen. The fuzzy observer design is divided in three phases:

1). PCA Analysis: The non-linear model is linearized around the operating points defined by the input variations (table 1). A PCA analysis is made to choose fuzzy variables
and the local models, which characterize in the best way the non-linear system.

2. Linear observers design: A classical fuzzy observer for each local model is designed.

3. Non-linear dynamic reconstruction: The local estimated states are interpolated using the Takagi-Sugeno algorithm explained in 3.1 to get the global estimated state.

So, a fuzzy observer rule example is shown below:

IF Q in is low AND S2 in is little AND pH is small

THEN

\[
\hat{x}_i = A \hat{x}_i + B u + K_i (y - \hat{y})
\]

The final estimated state is calculated with equations 20 and 21.

IV. RESULTS

A simulation study was made to test the observer performances.

The figure (6) shows the biomass and substrate (for the first and the fourth collocation points) estimation when an input variation is considered. The input flow rate changes from 4 to 6 lt/hr. To show the convergence, arbitrary initial conditions are chosen. Then, convergence is illustrate at the simulation begin. Biomass is perfectly estimated in the whole of simulation. The Q in step induces, on substrate estimation, a transient error, which is eliminated in steady state.

The observer sensitivity to noise is shown in the figure (7). A noise signal was added to outputs (methane and carbon dioxide) and a step in Q in is also considered. The observer filters the noise and state variables are well estimated. As in the previous case, biomass is perfectly estimated and substrate estimation presents a transient error (after input step), which is also eliminated in steady state.

The simulations of other realistic situations show that the observer is sensitive to model errors and to strong disturbances in the input variables. Now some researches are being developed to improve the observer performances in these cases. Otherwise, in almost all cases the biomass is well estimated and this is the principal observer objective in this paper.

V. CONCLUSIONS

A Takagi-Sugeno fuzzy observer for the anaerobic digestion process is proposed to estimate the variables hard to measure or the non-measurable variables. The principal component analysis allows getting a simplified data set composed by linear models to characterize the non-linear state. Forty-five local observers, which are selected by three fuzzy variables, compose the fuzzy observer: Q in, S2 in and pH. The state variables are estimated from biogas measures, which are typically known in the real processes. A fluidized bed reactor was considered to validate the proposed observer. The fuzzy observer offers a good compromise between the quality of the estimation and the difficulty to design. The observer disadvantages are the quantity of local models to guarantee a long validity region, the transient errors and the sensitivity to model errors, which is a frequent problem in real cases. To improve these situations, it is possible to use the PCA results to choose only one local observer and design a fuzzy mechanism adapting the gain observer in function of operating conditions. The real life implementation is another future work.

APPENDIX

TABLE II

PARAMETERS FOR THE CONSIDERED FLUIDIZED BED REACTOR
(VOLUME: 11 LITRES)

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<td>mol/lt</td>
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<tr>
<td>K_b</td>
<td>5.7e-7</td>
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</tr>
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<tr>
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<td>AU/mol S_2</td>
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<tr>
<td>Y_2</td>
<td>1.05</td>
<td>mol S_2/ mol CH_4</td>
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**COLLOCATION MATRIX OBTAINED WITH THE LAGRANGE POLYNOMIALS**

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<tr>
<th>Collocation matrix (I)</th>
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**REFERENCES**


