Block Decoupling by Precompensation Revisited
Michel Malabre and Jorge A. Torres-Muñoz

Abstract—The block decoupling problem by admissible dynamic pre-compensation for LTI systems is considered. Admissibility refers to the preservation of the class of controlled output trajectories, i.e. functional output controllability is concerned, which is more demanding than just pointwise output controllability. This problem has been solved by Hautus and Heyman, within a transfer function matrix approach. Different new equivalent solvability conditions in terms of controllability subspaces, transfer function matrices or matrix pencils are given. One of these conditions (expressed in the input space) is at the origin of new necessary and sufficient conditions for block decoupling by general precompensation (possibly non admissible and nonsquare), in the wider sense of Basile and Marro.

Index Terms—Block decoupling, controllability subspaces, linear time-invariant (LTI) systems, precompensation, transfer function matrices.

I. INTRODUCTION

Input–output decoupling problem has attracted the interest of the control community from the very beginning of the development of the linear systems theory. Actually, there is a huge body of theoretical results within the so-called geometric approach as well as in the transfer function matrix approach. In this sense, recommended accounts can be found in [4], [13].

Block decoupling amounts to finding a control law in such a way that on the compensated system subsets of inputs drive specified blocks of outputs without interacting with other output blocks. In order to avoid trivial controllers such as the zero one, driving requirements for the outputs must be imposed, such as pointwise or functional output controllability preservation between the system and the compensated system.

A dynamic precompensation scheme is considered here, which includes dynamic (static) state (measurement) feedback as particular cases.

An elegant solution to block decoupling has been proposed by Basile and Marro in [7] within the geometric approach that relies on the properties of some specific controllability subspaces of the system. Their solution is general in the sense that the requirement on the controller is the preservation of the output pointwise controllability. Later, Hautus and Heymann [1] gave a solution to the block decoupling problem by admissible precompensation that requires the preservation of the output functional controllability properties. Their solution is based on the notion of independent rational spaces generated by the row-blocks of the transfer function matrix.

The main aim of the present contribution is to bridge some gap between these two different approaches. We first derive some equivalent characterizations in the transfer function, in the geometric as well as in...
the matrix pencil approaches for the block decoupling problem by admissible precompensation. We then deduce new transfer function conditions for the existence of general (i.e., possibly nonadmissible) solutions, in the wider sense of Basile and Marro [7].

Section II is devoted to notations and the problem statements, while basic properties of row block independency are given in Section III. Main results are presented in Section IV. These are illustrated by anmissible precompensation. We then deduce new transfer function conditions. Finally, Section VI is devoted to concluding remarks.

II. PRELIMINARIES

A. Notations

We will consider linear time invariant (LTI), for which state–space representation is given by

\[ \begin{align*}
    \dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*} \tag{1} \]

with state \( x(t) \in \mathbb{R}^n \), input \( u(t) \in \mathbb{R}^m \) and output \( y(t) \in \mathbb{R}^r \). For the sake of simplicity we assume that \( B \) is monic. Note that \((A, B)\) is not required to be controllable.

An equivalent representation of system (1), in the matrix pencil setting, is the system matrix by Rosenbrock [5]

\[ P(s, C) = \begin{bmatrix}
sI - A & -B \\
-C & 0
\end{bmatrix} \tag{2} \]

Finally, the transfer function matrix representation of system (1) is expressed as

\[ T(s) = C(sI - A)^{-1}B \tag{3} \]

which is a strictly proper rational transfer function matrix. For convenience, it is considered that entries of \( T(s) \) belong to the field of rational functions \( \mathbb{R}(s) \), while \( R[s] \) will stand for the ring of polynomials. A rational function matrix is proper if it has a finite limit when its argument goes to infinity, in the case the limit goes to zero it is strictly proper. Hereafter \( rank(T(s)) \) will stand for the generic rank of \( T(s) \).

The notion of polynomial basis is useful for rational spaces, its degree is given by the sum of the column degrees, which in turn are the degree of the in polynomials. A minimal polynomial basis of a rational space is a polynomial basis of minimal achievable degree. Such a basis is unique modulo pre or post multiplication by unimodular matrices. A minimal basis has no singularities at any finite value of \( s \) and the leading coefficient matrix is of full-column rank (column reduced).

In what follows, a useful tool will be \( \mathbb{R}^+ \), the maximal output nulling controllability subspace contained in the kernel of \( C \) (see [13] and [8]). Controllability subspaces of system (1) may be related with suitable polynomial bases for the matrix pencil \( P(s, C) \). Let us recall a result by Warren and Eckberg [12] written for the case of output nulling subspaces.

**Theorem 1:** The subspaces \( \mathbb{R}_{n_1} \), \( \ldots \), \( \mathbb{R}_{n_m} \subseteq \mathbb{R}^+ \), with \( \mathbb{R}^+ = \mathbb{R}_{n_1} + \ldots + \mathbb{R}_{n_m} \), of dimension \( n_i \), are output nulling controllability subspaces (in ker \( C \)) if and only if there exist minimal polynomial bases of the right null space of \( P(s, C) \), namely \( x_i(s) \in \mathbb{R}^+[s] \); \( u_i(s) \in \mathbb{R}^m[s] \) (i.e., \( \mathbb{R}^+ \subseteq \mathbb{R}^m[s] \)) such that

1) \( \deg u_i(s) = k_i \) and \( \deg x_i(s) = k_i - 1 \), for some \( k_i \geq r_i \);  
2) \( x_i(s) = \sum_{i=1}^{k_i} x_{i,j}^{(0)} s^{i-1} \) then \( \mathbb{R}_{n_i} = span \{ x_{i,1}^{(0)}, x_{i,2}^{(0)}, \ldots, x_{i,k_i-1}^{(0)} \} \), for \( i = 1, \ldots, m \).

B. The Block Decoupling Problem by Precompensation

Consider the system \((A, B, C)\) with \( m \)-input channels and \( p \)-output channels. The output is partitioned into \( k \) blocks through the list of non negative integers \((p_1, p_2, \ldots, p_k)\) such that \( p_1 + \ldots + p_k = p \). Assume that \( m_i \geq k \). The system \((A, B, C)\) is said to be \((p_1, p_2, \ldots, p_k)\) decoupled if there exist positive integers \((m_1, m_2, \ldots, m_k)\) satisfying \( m_1 + \ldots + m_k = m \), such that \( T(s) \) has the block diagonal form

\[ T(s) = \begin{bmatrix}
    T_{11}(s) & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & T_{kk}(s)
\end{bmatrix} \tag{6} \]

where \( T_{ii}(s) \) is of dimension \( p_i \times m_i \).

The control objective is to decouple a non decoupled system with the help of proper precompensation control laws, i.e.

\[ u(s) = K(s)y(s) \tag{4} \]

with \( K(s) \) proper. This is the most general control law in the sense that any static or dynamic state, or measurement, feedback is equivalent to a precompensation action of the type (4). But there also exist general proper precompensators which are not of feedback type. In order to exclude trivial compensators such as \( K(s) = 0 \), one has to add some output controllability requirements. In [1], Hautus and Heymann introduced the notion of admissible compensators that satisfy

\[ rank(T(s))K(s) = rank(T(s)) \tag{5} \]

This condition is equivalent to the preservation of the class of controlled output trajectories. It thus requires that no loss of functional output control occurs through the decoupling process.

**Remark 1:** Precompensation allows to obtain the simplest block decoupling solvability conditions. Block decoupling by state feedback \( u = Fx + Gc \) has been widely analyzed providing a bunch of solvability conditions. Currently, state feedback is termed non-regular when \( G \) is accepted to be non square but of full column rank and dynamic when \( F = F(s) \) is a dynamic transfer. Both offer advantages to decouple, for instance, nonsquare systems.

In the regular case, the existence of a static (or dynamic) decoupling feedback may be concluded by looking at the equality between two sets of integers, namely the infinite zero structure of \( T(s) \) and the union of the infinite zero structures of \( T_i(s) \) (see, for instance, [2]). Block decoupling by precompensation is possible under a much weaker «rank condition», namely the rank of \( T(s) \) is equal to the sum of the row-block ranks of \( T_i(s) \), which is illustrated in Section V. This condition is obviously weaker since the rank of a system is just the number of elements in its infinite zero structure.

In the nonregular case, dynamic state feedback and precompensation solutions are available, (see [3]), while the block (or even row by row) decoupling problem by non regular static state feedback is still unsolved.

Precompensation is good for block decoupling because it provides the simplest block decoupling solvability conditions, which are also the necessary ones when seeking for state feedback solutions. Then, it can be viewed as a prerequisite before handling more demanding feedback solutions.

III. A SUMMARY ABOUT ROW BLOCK INDEPENDENCY

A \( p \times m \) transfer function matrix is said to be partitioned in row-blocks relatively to a given list of nonnegative integers \((p_1, p_2, \ldots, p_k)\) s.t. \( p_1 + p_2 + \ldots + p_k = p \) if \( T(s) \) is decomposed as follows:

\[ T(s) = \begin{bmatrix}
    T_{11}(s) & \cdots & T_{1k}(s) \\
    \vdots & \ddots & \vdots \\
    T_{k1}(s) & \cdots & T_{kk}(s)
\end{bmatrix} \tag{6} \]
with \(T_i(s)\) being \(p_i \times m\) (for \(i = 1, \ldots, k\)) and \(T_i'(s)\) denoting transposition of \(T_i(s)\).

**Definition 1:** The \(k\) row blocks of \(T(s)\) are called independent if 
\[
\text{rank } T_i(s) = \sum_{i=1}^k \text{rank } T_i'(s).
\]
In the sequel, we shall need certain matrices, denoted by \(T_i'(s)\), deduced from matrix \(T(s)\) by keeping all the row-blocks except the \(i\)th row-block, that is to say
\[
T_i'(s) = \begin{bmatrix} T_1'(s) & \cdots & T_{i-1}'(s) & T_{i+1}'(s) & \cdots & T_k'(s) \end{bmatrix}'.
(7)

Considering the transfer function matrix \(T(s)\) as a map acting from an input linear rational space \(\mathcal{U}(s)\) to an output linear rational space \(\mathcal{Y}(s)\). Suppose matrix \(T(s)\) is row-block partitioned as in (6), according to a given nonnegative list of integers \((p_1, p_2, \ldots, p_k)\). Let \(S_i(s)\) be the rational linear space spanned by the rows of block \(T_i(s)\), for \(i = 1, \ldots, k\).

**Definition 2:** The radical space of subspaces \(S_1(s), S_2(s), \ldots, S_k(s)\), denoted as \(\mathcal{S}(s)\), is defined by
\[
\mathcal{S}(s) := \bigcap_{i=1}^k \left( \sum_{i=1}^k S_i(s) \right).
(8)
\]
This gives an alternative characterization for block independency.

**Proposition 2:** The row block spaces \(S_1(s), \ldots, S_k(s)\) are independent if and only if their radical space \(\mathcal{S}(s)\) is identically zero.

The proof follows directly from the notion of radical (see Wonham [10, p. 240]).

**Proposition 3:** Let \(S_i(s)\) be a strictly proper transfer function matrix such that its row span (over \(R(s)\)) is \(\mathcal{S}(s)\), the radical of \(S_i(s)\) \(i = 1, \ldots, k\), then
\[
\ker S_i(s) = \sum_{i=1}^k \ker T_i'(s)
\]
with \(T_i'(s)\) defined in (7), \(i = 1, \ldots, k\).

The condition may be deduced by taking the orthogonal of the radical space \(\mathcal{S}(s)\).

**Corollary 4:** With the previous notations, the following statements are equivalent:
1) \(\text{rank } T_i(s) = \sum_{i=1}^k \text{rank } T_i'(s)\);
2) \(\hat{S}(s) = 0\);
3) \(\sum_{i=1}^k \ker T_i'(s) = \mathcal{L}(s)\).

**IV. MAIN RESULTS**

In this section, we will first recall the solvability condition given by Hautus and Heymann (1983) for the block decoupling problem by admissible dynamic precompensation [1] and give new equivalent conditions. We will also characterize the existence of general solutions that are possibly nonadmissible.

Let us first recall the classical result of Hautus and Heymann (1983), [1].

**Theorem 5:** There exists an admissible precompensator \(K(s)\) such that \(T(s)K(s)\) is block decoupled if and only if \(S_1(s), \ldots, S_k(s)\) are \(R(s)\) independent, where \(S_i(s)\) denotes the rational linear space spanned by the rows of block \(T_i(s)\), \(i = 1, \ldots, k\).

We can now propose alternative equivalent characterizations.

**A. Block Decoupling by Admissible Precompensation**

**Theorem 6:** Consider a linear system with its different descriptions and notations introduced in Section II-A. Then, the following statements are equivalent.
1) There exists an admissible precompensator \(K(s)\) such that \(T(s)K(s)\) is block decoupled.
2) \(\hat{S}(s) = 0\).
3) \(\text{rank } T(s) = \sum_{i=1}^k \text{rank } T_i'(s)\).
4) \(\sum_{i=1}^k \ker T_i'(s) = \mathcal{U}(s)\), where \(\mathcal{U}(s)\) is the rational input space.
5) \(\sum_{i=1}^k \ker P(s, \hat{C}_i) = \ker [sI - A - B]\).
6) \(\sum_{i=1}^k \hat{R}_i^* = (A| B)\), where \(\hat{R}_i^*\) is the supremal controllability subspace in \(\ker \hat{C}_i, i = 1, \ldots, k\) and \(A| B = \text{Im}[B \quad AB \quad \ldots \quad A^{n-1}B]\).

Proof: 1) \(\iff\) 2): From Theorem 5.
2) \(\iff\) 3) \(\iff\) 4): From Corollary 4.
4) \(\iff\) 5): Observe first that null spaces of system representations (2) and (3) are related by,
\[
\ker [P(s, C)] = \ker \begin{bmatrix} sI - A & -B \\ 0 & T(s) \end{bmatrix}
\]
Define \(L(s)\) as a rational basis of \(\ker T(s)\), then
\[
\ker [P(s, C)] = \text{span} \left\{ T(s) \right\}
\]
where \(T(s) = \begin{bmatrix} (sI - A)^{-1}B \\ I_{m \times m} \end{bmatrix}\).

The same can be established for any system \(T_i(s)\). Thus, by considering \(T_i(s)\) as a rational basis of \(T_i(s)\), one gets
\[
\sum_{i=1}^k \ker P(s, \hat{C}_i) = \sum_{i=1}^k \text{span} \left\{ \Gamma(s) \mathcal{L}_i(s) \right\}
\]
with \(\mathcal{L}_i(s) := \left[ \mathcal{L}_1(s) \mathcal{L}_2(s) \cdots \mathcal{L}_k(s) \right]\).

Now, condition 4) is equivalent to \(\mathcal{L}(s) = \mathcal{I}_{m \times m}\) and, thus
\[
\sum_{i=1}^k \ker P(s, \hat{C}_i) = \text{span} \left\{ \Gamma(s) \right\}
\]
\[
= \ker [sI - A - B].
\]

5) \(\iff\) 6): Consider minimal bases \(V_i(s)\) and \(\Phi_i(s)\) of \(P(s, \hat{C}_i)\) and \(\ker [sI - A - B]\), respectively. Then, corresponding to any \(V_i(s)\) there exist subspaces \(\hat{R}_i^*\), for \(i = 1, \ldots, k\), which sum clearly belongs to subspace \((A| B)\).

Now, by condition 5) one has,
\[
\text{span} \left[ V(s) \right] = \text{span} \left[ \Phi(s) \right]
\]
which means that \(V(s) = [V_1(s), \ldots, V_k(s)]\) is a polynomial basis of \(\ker [sI - A - B]\), but is not necessarily a minimal one. However, it can be reduced to \(\Phi(s)\) applying unimodular operations \(U(s)\) on the right of \(V(s)\) as follows
\[
V(s)U(s) = \Phi(s),
\]
this means that the sum of subspaces \(\hat{R}_i^*\) equals \((A| B)\), by virtue of Theorem 1.

6) \(\iff\) 5) Let \(V_i(s) = [X_1(s), U(s)]\) and \(\Phi_i(s) = [Z(s), W(s)]\) be minimal polynomial bases for \(P(s, \hat{C}_i)\) and \(\ker [sI - A - B]\), respectively. Suppose by contradiction that
\[
\text{span} \left[ V_1(s), \ldots, V_k(s) \right] \subset \text{span} \left[ \Phi(s) \right]
\]
which involves two possibilities; first, suppose
\[
\text{span} \left[ X_1(s), \ldots, X_k(s) \right] \subset \text{span} \left[ Z(s) \right]
\]
i.e., the sum of subspaces $\tilde{R}_i$ should be a proper subspace of $\langle A | B \rangle$ contradicting condition 6).

Consider now the alternative assertion, namely,

$$\text{span} \left\{ X_i(s), \ldots, X_k(s) \right\} = \text{span} \left[ Z(s) \right]$$

by their nature one has that

$$X_i(s) = (sI - A)^{-1}BU_i(s): \quad Z(s) = (sI - A)^{-1}BW(s)$$

which in turn implies that the spaces spanned by minimal bases $[U_1(s), \ldots, U_k(s)]$ and $W(s)$ are the same as far as matrix $(sI - A)^{-1}B$ can be viewed as an injective map. Then, with $\Gamma(s)$ defined as in (9)

$$\text{span} \left\{ \Gamma(s)[U_1(s), \ldots, U_k(s)] \right\} = \text{span} \left\{ \Gamma(s)W(s) \right\}$$

which is actually in contradiction with assumption (10).\(^1\)

Remark 2: Note that $\sum_{i=1}^k \tilde{R}_i = \langle A | B \rangle$ implies that $\tilde{R}_i + \ker C_i = \langle A | B \rangle + \ker C_i$, which is the necessary and sufficient condition given by Basile and Marro [7] for block decoupling by the most general proper precompensation laws.\(^1\)

Remark 3: It has to be noted that all of the equivalent conditions (except the last one) in Theorem 6 can be extended, without any difficulty, to more general cases. This is first true for proper (not necessarily strictly proper) systems, as described by (1) but with output equation given by $y(t) = Cx(t) + Du(t)$.

More generally, this can be extended to regular generalized systems (also called implicit, singular or descriptor), for which in (1) the left hand side of the first equation is $E\dot{x}(t)$ in place of $\dot{x}(t)$, with $E$ not necessarily invertible. Regular in this context means that $(sE - A)$ is a square full rank matrix. With this regularity assumption, the “generalized” system matrix (2) exists with $sE - A$ in place of $sI - A$, as well as the generalized transfer function matrix (3), with $T(s) = C(sE - A)^{-1}B$.

It is then quite easy to check that the implications between conditions 1) to 5) can be proved exactly in the same way. Note that the equivalence between 1) and 3) in this generalized case has been established by Kučera [4], in the early 80’s.

Remark 4: Concerning the geometric counterparts, even though ad hoc tools exist both for proper systems (see Morse [11]), and for generalized ones (see e.g. Malabre [6]) to handle with controlled or controllability subspaces, it appears that those geometric tools are a bit more intricate than in the strictly proper case, and their use in the decoupling problem are not so developed, specially in the generalized case. Let us just forecast that a good way to extend condition 6) in Theorem 6, appears to be by projection of condition 5), with $sE - A$ in place of $sI - A$.

We are ready to give the second main contribution, namely a transfer function equivalent to the geometric conditions given by Basile and Marro for the solution of block decoupling with a general (not necessarily admissible) proper precompensator.

B. Transfer Function Conditions for General Block Decoupling

Lemma 7: There exists a (general) proper precompensator, say $G_i(s)$, which block decouples $T(s)$ if and only if there exists a proper precompensator, say $G(s)$, which makes $T(s)G(s)$ decouplable by admissible precompensation.

It appears that the set of all possible $G(s)$ which make $T(s)G(s)$ decouplable by admissible precompensation can be characterized in a simple way.

\(^1\)This is in fact a trivial extension of their condition when $\langle A | B \rangle$ is not necessarily equal to $X$.

Lemma 8: $G(s): \mathcal{V}(s) \rightarrow \mathcal{U}(s)$ (monic) makes $T(s)G(s)$ decouplable by admissible precompensation, that is (thanks to Theorem 6) $G(s)$ satisfies

$$\sum_{i=1}^k \ker \left[ T_i(s)G(s) \right] = \mathcal{V}(s)$$

if and only if

$$\ker G(s) = \sum_{i=1}^k \ker T_i(s).$$

Proof: (11) is equivalent to the following: For any $v(s) \in \mathcal{V}(s)$ there exist $w_i(s) \in \ker [T_i(s)G(s)]$ such that $v(s) = \sum_{i=1}^k w_i(s)$.

Since $G(s)$ is monic, this is equivalent to: for any $G(s)v(s) \in \ker G(s)$ there exist $w_i(s) = G(s)v_i(s) \in \ker [T_i(s)]$ such that $G(s)v(s) = \sum_{i=1}^k w_i(s)$, which is (12).

We can now deduce from these two lemmas the following.

Theorem 9: Consider the $k$-row block structured system with transfer function matrix $T(s)$ and with $\bar{T}_1(s)$ and $\bar{T}_2(s)$ as defined in (6), (7). Then $T(s)$ is decouplable by general proper precompensation if and only if

1) $\dim \sum_{i=1}^k \ker T_i(s) \geq k$;
2) $T_i(s)(\sum_{i=1}^k \ker T_i(s)) \neq 0$, for all $i = 1, \ldots, k$.

Proof: Direct from the previous lemmas. Condition 1) means that $\dim \mathcal{V} \geq k$, while condition 2) avoids trivialities coming from zero row blocks.

Consider the following transfer function matrix $T(s)$, with $(p_1, p_2) = (2, 2)$, and for which no admissible solution exists (since vector $[0 \ 0 \ 1]$ belongs to $\mathcal{S}_1(s) \cap \mathcal{S}_2(s)$).

$$T(s) = \begin{bmatrix} T_1(s) \\ T_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 & \frac{1}{s+1} \\ \frac{1}{s+1} & 0 & 1 \\ 0 & \frac{1}{s+1} & 2 \end{bmatrix}$$

right kernels are given by

$$\ker \bar{T}_1(s) = [1 \ 0 \ 0]' \quad \ker \bar{T}_2(s) = [0 \ 1 \ 0]'$$

let us choose $\ker G(s) = [\ker \bar{T}_1(s), \ker \bar{T}_2(s)]$. Then $T(s)G(s)$ is block decoupled.

This example taken from [1] shows that the price to pay when accepting non admissible precompensators to achieve more general block decoupling is that losses of rank will appear inside some of the row blocks.

V. Example

In this section, Theorem 6 is illustrated. Let

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} x(t)$$
be a controllable state space description (1) whose transfer function matrix (3) is
\[
T(s) = \begin{bmatrix}
T_1(s) \\
T_2(s)
\end{bmatrix} = \begin{bmatrix}
s^{-1} & 0 \\
0 & s^{-2}
\end{bmatrix}.
\]

Note that row by row decoupling is not possible since the total rank of \(T(s)\) is 2 \(\leq p = 3\). Let us consider block decoupling by precompensation with output partition \((p_1, p_2) = (2, 1)\). Note also that block decoupling by state feedback is not possible, since the infinite structure of the blocks \(\{(s^{-1}), (s^{-2})\}\) is not equal to the infinite structure of the system \((s^{-1}, s^{-2})\), [2].

Consider \([M_1(s), M_2(s)] = [s^{-1} - 1, 0]\) a left kernel basis of \(T(s)\) partitioned according to \(T_1(s), T_2(s)\). Since \(M_i(s)/T_i(s) = 0, i = 1, 2, \mathbf{S}(s) = 0\) implying condition 2).

Observe that condition 3) is trivially fulfilled.

By direct computation on \(T(s)\) one has
\[\ker T_1(s) = \text{span} \begin{bmatrix} s^{-1} \\ -1 \end{bmatrix}, \quad \ker T_2(s) = \text{span} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.\]

Clearly, since these rational vectors are independent, they span the two dimensional input space \(\ell(s)\). This means that condition 4) is satisfied.

Now, by simple computations one finds that
\[\ker [sI - A - B] = \text{span} \left\{ v_1'(s), v_2'(s) \right\},\]
with \(v_1'(s) = [1 \quad s \quad 0 \quad s^2 \quad 0], v_2'(s) = [0 \quad 0 \quad 1 \quad s \quad 0 s^2].\)

On the other hand
\[\ker P(s, \tilde{C}_1) = \text{span} \left\{ v_1(s) - sv_2(s) \right\},\]
\[\ker P(s, \tilde{C}_2) = \text{span} \left\{ v_2(s) \right\},\]
hence, condition 5) is verified.

Finally, let us denote by \(\{e_1, e_2, e_3, e_4\}\) the canonical basis of \(\mathcal{X} \cong \mathbb{R}^4\). It turns out that,
\[
\tilde{R}_2^* = \ker \tilde{C}_1 = \text{span} \{e_1, e_4\} \quad \text{and} \quad \tilde{R}_1^* = \ker \tilde{C}_2 = \text{span} \{e_1, e_2 - e_3, e_4\},
\]
on the other hand \((A | B) = \mathcal{X} \). Then, condition 6) is meet since,
\[
\tilde{R}_1^* + \tilde{R}_2^* = (A | B).
\]

VI. CONCLUDING REMARKS

The block decoupling problem by precompensation has been revisited in this note. First, we have analyzed the solvability of the block decoupling problem by admissible precompensation. Indeed, geometric, transfer function matrix or matrix pencil conditions, which are equivalent to the famous Hautus and Heymann’s solvability condition [1], are given in Theorem 6. Note that admissibility refers to the preservation of the functional output controllability along the decoupling process.

For that, an interesting independency property in terms of the input space is given (see Corollary 4). This opened the door to solve the more general situation of (possible) nonadmissible precompensators (see Theorem 9). This is a new transfer function matrix equivalent of the Basile and Marro’s classical geometric condition [7]. Here, just output pointwise controllability preservation is required during the decoupling process.

Let us mention that no controllability assumption has been made on system (1). The interest comes from the simultaneous disturbance rejection and decoupling problem for which it is usually impossible to assume that the dynamics coming from the disturbance would be controllable by the control input.

With respect to the stability issue, Hautus and Heymann [1] showed that “using combined precompensation, the decoupling problem and the stability question are separate (independent) issues.” For generalized systems, Kučera [4] obtained the same condition. Then, the solvability conditions presented in this note are still valid for block decoupling with internal stability under the standard assumptions that the system is stabilizable and detectable. This separation property holds true not only when admissible precompensators are looked for, but also with general ones, as stated in [8].

Finally, let us note that all these results can easily be dualized to solve the dual problem of failure detection and isolation (see [9]) by dynamic post-compensation.

ACKNOWLEDGMENT

The authors would like to express their gratitude to anonymous reviewers whose comments allowed to improve the focus and presentation of this note.

REFERENCES