Neural identification based on sliding mode observer

Xiaou Li, Wen Yu

Abstract—In this paper, a new on-line neural identification method is presented. The identified nonlinear systems are partial-state measurement. Their inner states, parameters and structures are unknown. The design is based on the combination of a sliding mode observer and a neuro identifier. First, a sliding mode observer, which does not need any information of the nonlinear system, is applied to get the full states. Then a dynamic multilayer neural network is used to identify the whole nonlinear system. The main contributions of this paper are: (1) A new observer based identification algorithm is proposed; (2) A stable learning algorithm for the neuro identifier is given.

I. INTRODUCTION

Identification is one of the essential problems in control theory, especially when we have no complete model informations. A effective method is to use neural network, because neural network is a model-free identifier, i.e., the plant can be considered as a "black box". A good deal of researches demonstrate that neural networks indeed fulfill the promise of providing model-free identification [8], [14]. Neuro identifier could be classified as static (feedforward) and dynamic (recurrent) ones [14]. Multilayer perceptrons are implemented for the approximation of nonlinear function in the right side-hand of differential equation [9]. A continue version of multilayer neural networks was used to estimate the nonlinearities of robot manipulator [12]. In these paper, static neural networks are proposed. The main drawback of these networks is that the weight updating laws utilize information on the local data structures (local optima) and the function approximation is sensitive to the training dates [5].

Dynamic neural networks may successfully overcome these disadvantages because of feedback structure [8]. Dynamic neural networks was first introduced by Hopfield [7] and then studied in [4], [15], [5] etc. There are two general concepts of recurrent training. Fixed point learning is aimed at making the neural networks reach the prescribed equilibria at perform steady-state matching [16]. Trajectory learning trains the networks to follow the desired trajectory in time [17]. Most of dynamic neural networks have no hidden layers [15], [20]. As single layer perceptrons, the approximation capabilities of these networks are limited. To overcome these shortcomings, a high-order dynamic neural network was proposed in [11]. It contains multiple nonlinear functions in order to approximate the complex nonlinear dynamics, the learning law is similar to the single layer case [15]. Another method is to employ multilayer dynamic neural networks, similar with the multilayer perceptrons the identification ability should be improved. To the best of our knowledge, there are few published results on multilayer dynamic neural networks. In this paper we will use multilayer dynamic neural networks to identify nonlinear system.

Most of works on identification use the assumption of the complete accessibility of the states, but in reality this is not always valid. In the case of only input and output are measurable, a complex neural network can match the input-output pairs very well. If the nonlinear system has many inner states, the input-output mapping is not enough to model the whole nonlinear system. To overcome this shortcoming, there exist at least two ways:

To use neural networks which contain a Luenberger-like observer [10]. The dimension of neural networks is the same as the plant. Since only output error is available, the weights which are corresponding to the inner states cannot be changed by the identification error.

To employ "separation principle", i.e., states observer and system identification are treated separately. The difficult is that model-based observers cannot be used, because no exact knowledge of the plant are available. Model-free observers, such as high-gain observer [13] and sliding mode observer [1] may be useful, but they are suitable for special plant. For example, high-gain observer requires the nonlinear plant has the linearization form, the sliding mode observer [1] is suitable for robot dynamic.

The combination of nonlinear observer with neural network is a good direction to improve the identification accuracy. In this paper sliding mode observer will be applied to estimate the inner states. Compared with [1], we do not need any information of the plant, we only assume the nonlinear function is bounded. By means of a Lyapunov-like analysis we derive stability conditions of the observer and the weights updating law of multilayer dynamic neural networks. The simulation results show that the observer-based neuro identifier may improve the identification accuracy effectively.

II. SLIDING MODE OBSERVER

Generally a MIMO nonlinear system can be written as

\[ x_t = f(x_t, u_t), \quad y_t = Cx_t \]  \hspace{1cm} (1)

where \( x_t \in \mathbb{R}^n \) is the state of the plant. \( u_t \in \mathbb{R}^m \) is bounded control input which may stabilize the nonlinear system (1), \( |u_t| \leq \overline{u}, \ y_t \in \mathbb{R}^l \) is measurable output, \( C \) is known output matrix. Transform the system (1) into a normal form:

\[ \dot{x}_t = Ax_t + F(x_t, u_t), \quad y_t = Cx_t \]  \hspace{1cm} (2)
where \( F(x_t, u_t) := f(x_t, u_t) - Ax_t \), \( A \) is a special matrix such that the pair \((A, C)\) is observable.

Let construct the sliding mode observer as

\[
\begin{align*}
\dot{x}_t &= A\hat{x}_t + S(\hat{x}_t, e) - Ke_t \\
\dot{e}_t &= C\hat{x}_t
\end{align*}
\]  
(3)

where \( e_t \) is output error defined as:

\[
e_t := y_t - y_t = C\Delta_t = C(\hat{x}_t - x_t),
\]
\( \Delta_t \) is defined as observer error. The sliding mode term \( S(\hat{x}_t, e_t) \) is selected as

\[
S(\hat{x}_t, e_t) = \frac{-P^{-1}C^T\Delta_t}{||C\Delta_t||} = -\rho P^{-1}C^T \text{sign}(e_t)
\]
(4)

where \( \rho \) is positive constant.

Clearly the sliding mode observer (3) is not depended on the nonlinear plant (1).

The derivative of observer error is

\[
\Delta_t = A\Delta_t + S(\hat{x}_t, e_t) - F(x_t, u_t) = A_0\Delta_t + S(\hat{x}_t, e_t) - F(x_t, u_t)
\]
(5)

where \( A_0 := A - KC \). Because \((A, C)\) is observable, there exists \( K \) such that \( A_0 \) is stable. So the following Lyapunov equation has a positive solution \( P \) such that \( A_0^TQ + QA_0 = -Q \), \( Q = Q^T > 0 \) for some positive definite matrix \( Q \).

Let us assume the nonlinear function \( f(x_t, u_t) \) satisfies following boundness assumption

\( A1: \)

\[
f(x_t, u_t) - Ax_t = -P^{-1}h(x_t, u_t)
\]

where \( h(x_t, u_t) \) is bounded a function as:

\[
\left( CC^T \right)^{-1} \left( ||C|| ||h(x_t, u_t)|| \right) < \rho, \quad \rho > 0.
\]

where \( \text{det} \left( CC^T \right) \neq 0 \)

\( \textbf{Theorem 1:} \) Under the assumption \( A1 \), the observer error between the sliding mode observer (3) and nonlinear system (1) is asymptotically stable

\[
\lim_{t \to \infty} \Delta_t = 0
\]

\( \textbf{Proof:} \) Let consider the following Lyapunov function candidate:

\[
V_t = \Delta_t^T P \Delta_t
\]

Calculate its derivative

\[
V_t = \Delta_t^T (A_0^T P + PA_0) \Delta_t + 2\Delta_t^T P S(\hat{x}_t, e_t) - F(x_t, u_t)
\]

Using \( A1 \)

\[
F(x_t, u_t) = -P^{-1}h(x_t, u_t)
\]

If we select \( S(\hat{x}_t, e_t) \) as \( (4) \), we have

\[
V_t = -\Delta_t^T Q\Delta_t + 2\Delta_t^T h(x_t, u_t) - 2\Delta_t^T C^T \text{sign}(e_t)
\]

\[
= -\Delta_t^T Q\Delta_t + 2\Delta_t^T h(x_t, u_t) - 2 ||\Delta_t|| \rho
\]

\[
= -\Delta_t^T Q\Delta_t + 2 \left( CC^T \right)^{-1} ||C|| ||\Delta_t|| ||h(x_t, u_t)|| - 2 ||\Delta_t|| \rho
\]

\[
= -\Delta_t^T Q\Delta_t + 2 ||\Delta_t|| \left[ \left( CC^T \right)^{-1} ||C|| ||h(x_t, u_t)|| - \rho \right] < 0
\]

Since \( V_t < 0, \Delta_t \in L \). From the error equation (5) we also conclude that \( \Delta_t \in L \). Because \( V_t \leq -\Delta_t^T Q\Delta_t \) and \( V_t \) is bounded process, \( \Delta_t \) is quadratically integrable and bounded \( \Delta_t \in L^2 \). Using Barbalat’s Lemma [14] we obtain that the observer error \( \Delta_t \) is asymptotically stable, so \( \lim_{t \to \infty} \Delta_t = 0 \).

\( \textbf{Remark 1:} \) The assumption \( A1 \) is easy to be satisfied if \( f(x_t, u_t) \) is bounded, because \( A, P \) and \( h(x_t, u_t) \) are selected by the user. Many systems have bounded nonlinear function \( f(x_t, u_t) \), for example the mechanical systems. The design procedure of sliding mode observer (3) is shown in Fig.1.

\section{III. OBSERVER-BASED NEURO IDENTIFIER}

Form input \( u_t \) and output \( y_t \) one can identify nonlinear system (1) or (2) via neural networks, but this neuro model only reflects the relation of input and output. Last section give us an asymptotic estimation of inner states, if the estimated states \( \hat{x}_t \) in (3) is used to identify the system, the neural network model will approximate the whole nonlinear system. Let us consider the following multilayer dynamic neural network to identify this nonlinear system (1)
where \( \forall t \in [0, \infty) \), the vector \( \tilde{x}_t \in \mathbb{R}^n \) is the state of the neural network. \( \mathbf{A}_s \in \mathbb{R}^{m \times n} \) is a stable matrix which will be specified after. The matrices \( \mathbf{W}_{1,t}, \mathbf{V}_{1,t} \in \mathbb{R}^{m \times n} \), \( \mathbf{V}_{2,t} \in \mathbb{R}^{n \times m} \) and \( \mathbf{V}_{2,t} \in \mathbb{R}^{n \times n} \) are the weights describing output and hidden layers connections. \( \sigma(.) \in \mathbb{R}^m \) is sigmoidal vector functions, \( \phi(.) \) is \( \mathbb{R}^{m \times m} \) diagonal matrix

\[
\phi(.) = \text{diag} \{ \phi_1(V_{2,t} \tilde{x}_t), \ldots, \phi_m(V_{2,t} \tilde{x}_t) \}.
\]

The elements of \( \sigma(.) \) (as well as \( \phi(.) \)) are selected as sigmoid functions

\[
\sigma_i(x) = a_i / \left(1 + e^{-b_i x}\right) - c_i.
\]

The single layer dynamic neural networks [15], [20] are the special case of (7) when

\[
m = n \quad \text{and} \quad \mathbf{V}_1 = \mathbf{V}_2 = \mathbf{I}.
\]

Now the identification goal is to make the states of neural network (7) match the states of the real plant. Therefore we select the identification goal as: minimize the term

\[
J_{\text{min}} = \min_{\mathbf{W}} J \quad J = \|x - \tilde{x}\|^2 \quad \text{R}.
\]

So, for any \( \eta > 0 \), we have

\[
J \leq (1 + \eta) \|x - \tilde{x}\|^2 \quad \text{R} + (1 + \eta^{-1}) \|x - \tilde{x}\|^2 \quad \text{R}.
\]

The minimum of the term \( \|x - \tilde{x}\|^2 \) has already been solved in the previous section. If we select \( \mathbf{R} = (1 + \eta^{-1}) \mathbf{I}_1 \), we can reformulate the identification goal as: minimize the term \( \|x - \tilde{x}\|^2 \). In fact, this is Separation Principle.

Let us define the identification error as

\[
\Delta_t := x_t - \tilde{x}_t.
\]

Because \( \sigma \) and \( \phi \) are chosen as sigmoid functions, the following general Lipschitz conditions are fulfilled:

\[
\begin{align*}
\tilde{\sigma}_1^T \Lambda_1 \Delta_t & \leq \Delta_t^T \Lambda_2 \sigma \Delta_t, \\
\tilde{\phi}_1(u_t) & \leq \bar{u} \tilde{\sigma}_2^T \Lambda_3 \Delta_t, \\
\tilde{\phi}_2(u_t) & \leq \bar{u} \tilde{\sigma}_2^T \Lambda_3 \Delta_t.
\end{align*}
\]

where

\[
\begin{align*}
\tilde{\sigma}_1 := & \sigma(V_1 \tilde{x}_t) - \sigma(V_2 \tilde{x}_t), \\
\tilde{\phi}_1 := & \phi(V_2 \tilde{x}_t) - \phi(V_2 \tilde{x}_t), \\
\tilde{\phi}_2(u_t) := & \phi(V_2 \tilde{x}_t) - \phi(V_2 \tilde{x}_t)u_t.
\end{align*}
\]

Generally the neural network (7) can not match the given nonlinear system (1) exactly, the nonlinear system (1) can be represented as

\[
\tilde{x}_t = A_n \tilde{x}_t + W_1^* \sigma(V_1 \tilde{x}_t) + W_2^* \phi(V_2 \tilde{x}_t)u_t + \tilde{f}_t.
\]

where \( \tilde{f}_t \) is modelling error, \( W_1^*, W_2^*, V_1^* \), and \( V_2^* \) are given constant matrices.

If a bounded control input \( u_t \) may stabilize the nonlinear system (1), the unmodeled dynamics \( \tilde{f}_t \) is bounded [12]. Let us assume that

\[
\textbf{A2: There exists positive constant } \bar{\eta} \text{ such that}
\]

\[
\left \| \tilde{f}_t \right \|^2 \bar{\eta} \leq \bar{\eta} \Lambda_t^T \Lambda_t \leq \bar{\eta}, \quad \Lambda_t = \Lambda_t^T > 0
\]

It is well known [18] that if the matrix \( \mathbf{A}_s \) is stable, the pair \( (\mathbf{A}_s, \mathbf{R}^{1/2}) \) is controllable, the pair \( (\mathbf{Q}^{1/2}, \mathbf{A}_s) \) is observable, and the special local frequency condition or its matrix equivalent

\[
\mathbf{A}_s^T \mathbf{R}^{-1} \mathbf{A}_s - \mathbf{Q} \geq \frac{1}{4} \left[ \mathbf{A}_s^T \mathbf{R}^{-1} - \mathbf{R}^{-1} \mathbf{A}_s \right]^T
\]

is fulfilled, then the matrix Riccati equation

\[
\mathbf{A}_s^T \mathbf{P} + \mathbf{PA}_s + \mathbf{PRP} + \mathbf{Q} = 0
\]

has a positive defined solution \( \mathbf{P} \). So it is reasonable to introduce the following assumption:

\[
\textbf{A3: For a given stable matrix } \mathbf{A}_s \text{, there exists a strictly positive defined matrix } \mathbf{Q}_1 \text{ such that the matrix Riccati equation (15) with}
\]

\[
\mathbf{R} = 2 \mathbf{W}_1 + 2 \mathbf{W}_2 + \Lambda_1^{-1}, \quad \mathbf{Q} = \mathbf{Q}_1 = \mathbf{A}_s + \bar{u}^2 \mathbf{A}_s \]

where \( \mathbf{W}_1 := \mathbf{W}_1^T \Lambda_1^{-1} \mathbf{W}_1^T, \mathbf{W}_2 := \mathbf{W}_2^T \Lambda_2^{-1} \mathbf{W}_2^T \) has a positive solution \( \mathbf{P} \).

The another main contribution of this paper is that we give a new on-line learning law for the multilayer dynamic network (7):

\[
\begin{align*}
\mathbf{W}_{1,t} &= -2s_t K_1 P \Delta_t \sigma_t^T - 2s_t K_2 P \Delta_t \tilde{V}_{1,t}^T D_\sigma, \\
\mathbf{W}_{2,t} &= -2s_t K_2 P \Delta_t (\phi(u_t))^T - 2s_t K_2 P \Delta_t \tilde{V}_{2,t}^T D_\sigma, \\
\mathbf{W}_{1,t} &= -2s_t K_4 P \tilde{V}_{1,t}^T D_\sigma e_t^T - s_l K_3 A_2 \tilde{V}_{1,t}^T \tilde{V}_{2,t}, \\
\mathbf{W}_{2,t} &= -2s_t K_4 P \tilde{V}_{2,t}^T D_\sigma e_t^T - s_l K_4 A_2 \tilde{V}_{2,t} \tilde{V}_{2,t}.
\end{align*}
\]

where

\[
\begin{align*}
\Lambda_1, \Lambda_2, \Lambda_3, \text{ and } \Lambda_4 \text{ are } \text{positive define matrices}
\end{align*}
\]

\[
\begin{align*}
\mathbf{K}_i & \in \mathbb{R}^{n \times n} (i = 1 \ldots 4) \text{ are positive defined matrices, } \\
\text{P is the solution of the matrix Riccati equation given by (15). Because } \mathbf{W}_1, \mathbf{W}_2, \tilde{V}_1, \text{ and } \tilde{V}_2 \text{ can be any constant matrices, we select them as the initial conditions:}
\end{align*}
\]

\[
\begin{align*}
\mathbf{W}_{1,0} &= \mathbf{W}_1, \quad \mathbf{W}_{2,0} = \mathbf{W}_2, \quad \mathbf{V}_{1,0} = \mathbf{V}_1, \quad \mathbf{V}_{2,0} = \mathbf{V}_2.
\end{align*}
\]

The following theorem states the fact that the learning law suggested above turns out to be globally stable.

\[
\textbf{Theorem 2: Let us consider a nonlinear system (1) and multilayer dynamic neural network (7) whose weights are}
\]
adjusted as (17). If the assumptions A2 and A3 are held, we can conclude that

(I) The weights of neural networks $W_{i,t}, \bar{V}_{i,t}$ and identification error $\Delta_t$ are bounded.

(II) For any $T \in (0, \infty)$ the identification error $\Delta_t$ converges to the residual set

$$ \mathcal{D}_{\alpha_t} = \{ \Delta_t \mid \| \Delta_t \|_{Q_1} \leq \bar{\eta} \} $$

**Proof:** Using (7) and (13), the dynamic of identification error is

$$ \dot{\Delta}_t = A_0 \Delta_t + \bar{W}_1 \sigma_t + \bar{W}_2 \Phi_t + \bar{W}_3 \Phi_t' + \bar{f}_t $$

(21)

Define Lyapunov function candidate as: if $\| \Delta_t \|_{Q_1} \leq \bar{\eta}$

$$ V_{1,t} = \max (PQ_t^{-1}) \bar{\eta} + \frac{1}{2} \left[ \bar{W}_{1,t} K_1^{-1} \bar{W}_{1,t}^T \right] $$

$$ + \frac{1}{2} tr \left[ \bar{W}_{2,t} K_2^{-1} \bar{W}_{2,t}^T \right] + \frac{1}{2} tr \left[ \bar{V}_{1,t} K_3^{-1} \bar{V}_{1,t}^T \right] $$

$$ + \frac{1}{2} tr \left[ \bar{V}_{2,t} K_4^{-1} \bar{V}_{2,t}^T \right] $$

(22)

Because $\bar{W}_{i,t} = W_{i,t}$, from the dead-zone learning law (17), the derivative of $V_t$ is: if $\| \Delta_t \|_{Q_1} \leq \bar{\eta}$

$$ V_{1,t} = 0 $$

if $\| \Delta_t \|_{Q_1} > \bar{\eta}$

$$ V_{2,t} = 2 \Delta_t^T P A_t + \frac{1}{2} tr \left[ \bar{W}_{1,t} K_1^{-1} \bar{W}_{1,t}^T \right] $$

$$ + \frac{1}{2} tr \left[ \bar{W}_{2,t} K_2^{-1} \bar{W}_{2,t}^T \right] + \frac{1}{2} tr \left[ \bar{V}_{1,t} K_3^{-1} \bar{V}_{1,t}^T \right] $$

$$ + \frac{1}{2} tr \left[ \bar{V}_{2,t} K_4^{-1} \bar{V}_{2,t}^T \right] $$

(23)

Now let us discuss the term $2 \Delta_t^T P A_t$, according to (21)

$$ 2 \Delta_t^T P A_t = 2 \Delta_t^T P A_t \Delta_t $$

$$ + 2 \Delta_t^T \bar{W}_1 \bar{\sigma}_t + 2 \Delta_t^T \bar{W}_2 \bar{\Phi}_t $$

$$ + 2 \Delta_t^T \bar{W}_3 \bar{\Phi}_t' + \bar{f}_t $$

(24)

The following matrix inequality [20] is used to estimate the right side of (24)

$$ X^T Y + (X^T Y)^T \leq X^T A X + Y^T A Y $$

(25)

which is valid for any $X, Y \in \mathbb{R}^{n \times k}$ and for any positive defined matrix $0 < A = A^T \in \mathbb{R}^{n \times n}$.

Using (12) we have

$$ \Delta_t^T P \bar{W}_1 \bar{\sigma}_t \leq \Delta_t^T P \bar{W}_1 \Delta_t + \bar{\sigma}_t^T A_t \Delta_t $$

$$ \leq \Delta_t^T (P W_1 P + \Lambda) \Delta_t $$

$$ \Delta_t^T P \bar{W}_2 \bar{\Phi}_t' \leq \Delta_t^T (P W_2 P + \Lambda) \Delta_t $$

$$ \Delta_t^T P \bar{W}_3 \bar{\Phi}_t \leq \Delta_t^T (P W_3 P + \Lambda) \Delta_t $$

(27)

$$ 2 \Delta_t^T P \bar{f}_t $$

$$ \leq 2 \Delta_t^T P \bar{f}_t + \bar{f}_t^T A_t \bar{f}_t $$

$$ + 2 \Delta_t^T P \bar{W}_2 \bar{\Phi}_t \bar{V}_{2,t} $$

$$ + 2 \Delta_t^T P \bar{W}_3 \bar{\Phi}_t' \bar{V}_{2,t} $$

(28)

From A2, $2 \Delta_t^T P \bar{f}_t$ can be estimated as

$$ 2 \Delta_t^T P \bar{f}_t \leq \Delta_t^T P \bar{f}_t + \bar{f}_t^T A_t \bar{f}_t $$

$$ \leq \Delta_t^T Q_1 \Delta_t + \bar{\eta} $$

(29)

Using (24), (26) and (27), $V_{2,t}$ can be rewritten as

$$ V_{2,t} \leq - \Delta_t^T Q_1 \Delta_t + \bar{\eta} $$

(30)

It is noted that when

$$ \| \Delta_t \|_{Q_1} > \bar{\eta} $$

$$ V_{2,t} < 0, \forall t \in [0,T], $$

Since $V_{1,t} = 0$ and $V_{2,t} < 0$, $V_t$ is bounded, that is (1).

So the total time during which $\| \Delta_t \|_{Q_1} > \bar{\eta}$ is finite. Let $T_k$ denotes the time interval during which $\| \Delta_t \|_{Q_1} > \bar{\eta}$

- If only finite $T_k$ then the sequence $\| \Delta_t \|_{Q_1}$ will eventually stay inside of this circle.
- If $\| \Delta_t \|_{Q_1}$ leave the circle infinite times, since the total time $\| \Delta_t \|_{Q_1}$ leave the circle is finite,

$$ \sum_{k=1}^{\infty} T_k < \infty, \lim_{k \to \infty} T_k = 0 $$

(30)

So $\| \Delta_t \|_{Q_1}$ is bounded via an invariant set argument.

From (21) $\Delta_t$ is also bounded. Let $\| \Delta_t \|_{Q_1}$ denote the largest tracking error during the $T_k$ interval. Then (30) and bounded $\| \Delta_t \|_{Q_1}$ imply that

$$ \lim_{k \to \infty} \left[ \| \Delta_t \|_{Q_1} - \bar{\eta} \right] = 0 $$

(31)

So $\| \Delta_t \|_{Q_1}$ will convergence to $\bar{\eta}$. (II) is achieved.
IV. SIMULATIONS

Example 1: Let us consider a two-links robot manipulator, its dynamics of which can be expressed as follows [12]:

\[
M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) + f_D(q) = \tau
\]

where \( q = [q_1, q_2]^T \) consists of the joint variables, \( \tau \in \mathbb{R}^2 \) is the generalized forces, \( M(q) \) is the inertia matrix, \( V(q, \dot{q}) \) is centripetal-Coriolis matrix, \( G(q) \) is gravity vector, \( f_D(q) \) is the friction vector. \( M(q) \) represents the positive defined inertia matrix. For the case of two links, the elements can be represented as:

\[
M(q) = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\]

The design steps are shown as in Fig. 1. We can select

\[
A_0 = 10, \quad B = 0.4 
\]

The above boundary layer controller offers a continuous approximation to the discontinuous sliding mode inside the boundary layer and guarantees the observer error within any neighborhood of the origin [2]. One can see that the observer-based identifier is better than a general identifier when partial-state are measurable.

No loss of generality, we select \( \|h(x_t, u_t)\| < \rho, \quad \rho = 10 \). To eliminate chattering, the following boundary layer compensator can be used:

\[
S(k_1, e_1) = -P_-C^T \Delta \rho
\]

where \( \Delta \) is the friction vector. For the two links manipulator we use two observers: observer (a):

\[
\begin{align*}
\dot{q}_a &= Aq_a + S(q_a, e) - Ke_t \\
\dot{e}_a &= Cq_a, \quad q_a := [q_1, q_1]
\end{align*}
\]

observer (b):

\[
\begin{align*}
\dot{q}_b &= Aq_b + S(q_b, e) - Ke_t \\
\dot{e}_b &= Cq_b, \quad q_b := [q_2, q_2]
\end{align*}
\]

The initial conditions are \( q(0) = \hat{q}(0) = 0 \). The weights are updated according to (17) with \( K = 2, \dot{A} = 0 \). The identification results for \( q_1 \) are shown in Fig. 2. If we do not use observer, the neuro identifier as in (37) will be two dimension, we select \( \hat{Q} = [\hat{q}_1, \hat{q}_2, \hat{q}_3] \). We select \( W_{1,t} \) and \( W_{2,t} \in R^{4 \times 3}, \quad V_{1,t} \) and \( V_{2,t} \in R^{3 \times 4} \),

\[
\begin{align*}
\hat{q}_1 &= A_n\hat{q} + W_{1,t}o(V_{1,t}\hat{q}) + W_{2,t}\phi(V_{2,t}\hat{q})
\end{align*}
\]

\[
\begin{align*}
\hat{q}_1 &= [\hat{q}_1, \hat{q}_2, \hat{q}_3] \quad \text{and} \quad \hat{q}_1 = [\hat{q}_1, \hat{q}_2, \hat{q}_3]
\end{align*}
\]

The initial conditions are \( W_{1,t} = W_{2,t} = V_{1,t} = V_{2,t} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \), \( q(0) = \hat{q}(0) = 0 \). The weights are updated according to (17) with \( \dot{p} = 0.2, \quad K_t = 10, \dot{A} = -2 \). The identification results for \( \hat{q}_1 \) are shown in Fig. 2. If we do not use observer, the neuro identifier as in (37) will be two dimension, we select \( \hat{Q} = \begin{bmatrix} \hat{q}_1, \hat{q}_2, \hat{q}_3 \end{bmatrix} \). We can see that the observer-based identifier is better than a general identifier when partial-state are measurable.

Fig. 2. Observer-identifier for \( q_1 \)
We extend the nonlinear identification technique to the more general cases: (1) the system is black box, (2) only input-output are available. Because the system is black box, the multilayer dynamic neural networks are used; because only input-output are measurable, the sliding mode observer is given. The combination of these two methods can be easily extended to nonlinear output feedback control.

REFERENCES


