CONTROL OF NUCLEAR RESEARCH REACTORS BASED ON A GENERALIZED HOPFIELD NEURAL NETWORK

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ABSTRACT—The purpose of this paper is to present a solution to the minimization problem of the transient time to accomplish the switching between different levels of power in a nuclear research reactor satisfying the inverse period constraint and avoiding to use any physical model of the plant. The strategy here proposed consists of two stages. first, the optimal trajectory which satisfies the constraint is calculated off-line; second, a control law based on a generalized Hopfield neural network is employed to assure that the reactor power follows this optimal trajectory. The boundedness for both the weights and the identification error is guaranteed by a new online learning law. Likewise, proposed control law guarantees an upper bound for the tracking error. The effectiveness of this procedure is illustrated by numeric simulation.

Key Words: Model-free control. Hopfield neural network. Nuclear research reactor

1. INTRODUCTION

The basic control problem of any nuclear research reactor consists of increasing or decreasing the neutron power from a certain level \( R_0 \) to a new desired level \( R_f \) and maintaining the reactor stable at the new power level. This control task is achieved by partially extracting the control rods from the reactor’s core to increase power or partially inserting these rods into the core to reduce power (see Figure 1). However, this task can not be accomplished arbitrarily since, due to a regulatory requirement, the constraint of the inverse period must be always satisfied. If this constraint were not complied, a scram (automatic shut down of the reactor) would be immediately activated. Evidently, scrams provoke a bothering interruption in the normal operation of the reactor. Consequently, it is convenient to avoid them in order to increase reactor availability. Notwithstanding, at the same time, to achieve this last objective, the transient time of switching between different levels of power should be minimized. Currently, the most of nuclear research reactors are controlled by a PID-type scheme in which the control action has been conveniently
limited. Nevertheless, the application of this procedure does not permit to obtain the possible smallest transient time. Besides, with the purpose of maintaining acceptable system performance, PID parameters must be frequently tuned. Given the features of the aforementioned constrained control problem, it could be logic to suggest the direct application of the classical optimal control theory as a first attempt to find a solution. However, a control system which uses directly on-line Pontryagin’s maximum principle or Bellman’s dynamic programming could be very costly from a computational point of view. Besides, these techniques tolerate neither modeling errors nor unforeseen perturbations. To overcome these drawbacks, in [1], Aleksakov proposed to use an indirect strategy. In that work, Aleksakov established that the unique optimal solution to the problem of minimizing the transient time for switching between two different levels of power satisfying at the same time the inverse period constraint consists of achieving the control action in two stages: 1) During the transient stage, the instantaneous inverse period of the reactor must be maintained equal to its upper bound. 2) Starting from the time in which the specified power level is attained, the neutron power must remain constant. Using this principle, Aleksakov developed a controller based on the physical model of the reactor. In a completely independent way and using a novel approach, in [2], the same principle of the Aleksakov’s work was found. With this principle, Bernard developed the MIT-SNL Period-Generated Minimum Time Control Laws approach. Nevertheless, the main drawback associated to these two works is their reliance on the physical model. It is well known that the dynamic behavior of a nuclear reactor is depending on many factors and its precise modeling from first principles implies necessarily the employment of high order nonlinear system. To reduce the complexity of these models, several assumptions must be accomplished. Consequently, the applicability of the mentioned works is limited to only certain very restricted conditions both physical and temporal.
To extend the previous results and overcome the dilemma between precision and complexity associated to the first principles modeling, in [3] it was suggested to use an associative stochastic automaton for controlling reactor power ascent. Although this system can completely dispense with the physical model, nonetheless, its performance is inferior with respect to Bernard's controller. In addition, a long previous training process is required. On the other hand, a different strategy was employed in [4] where was developed a variation of a Mandani controller which was applied to a nuclear reactor. Although this controller satisfies, at least in simulation, the inverse period constraint of the reactor and does not show excessive overshoots, its main drawback lies on the absence of a rigorous proof of its stability. Another inconvenience is related to its robustness properties, which have not been exhaustively tested. Apart from fuzzy logic, the artificial neural networks have also been proposed to control nuclear reactors with a partially (or inclusive totally) incomplete information about plant model [5-9]. In these works, generally the networks employed have been static or discrete multilayer recurrent with a long preliminary off-line learning phase or else, they lack a rigorous proof of the stability of the corresponding closed-loop system based on the ANN. Besides, many of the mentioned works have been developed to power reactors. Consequently, the structure of these networks results unnecessarily complex when they are applied to nuclear research reactors. Thereby in this paper, we propose a strategy based on a differential neural network to obtain the switching between two different levels of power in a minimum time. To accomplish this task, first, the optimal trajectory that satisfies the constraint of the inverse period is calculated off-line without considering any nuclear dynamics. Next, to obtain a mathematical description of the nuclear behavior, the identification of the plant is accomplished by a generalized Hopfield neural network using only the external reactivity and the neutron power. This procedure is justified because the internal nuclear dynamics is stable. Thus, the use of any observer is conveniently avoided and the design process is simplified. Based on this neural model, the plant is forced to follow the optimal trajectory by an appropriate control law. This strategy is tested by numeric simulation showing satisfactory results. The main contributions of this paper are: a) a new learning law which permits to guarantee the boundedness for both the weights and the identification error, b) a control law which permits to establish an upper bound for the tracking error. Besides, to the best of our knowledge, this is the first time that a Hopfield neural network is used to control a nuclear reactor.

2. CALCULUS OF THE OPTIMAL TRAJECTORY SUBJECTED TO INVERSE PERIOD CONSTRAINT

The instantaneous inverse period of a nuclear reactor can be defined as

$$ \omega_t := \frac{\dot{n}_t}{n_t} $$

where $n_t$ is the neutron power. The constraint of the inverse period consists of imposing an upper bound on (1), that is,

$$ \omega_t \leq \frac{1}{T} $$

where $T$ is the particular nominal period for each nuclear reactor. As it was mentioned in section I, we want to minimize the transient time to accomplish the commutation between different levels of power in a nuclear research reactor satisfying the inverse period constraint (2) but avoiding the use on-line of maximum principle or dynamic programming. To accomplish this task, we propose first to calculate off-line the optimal trajectory that satisfies the constraint (2).
Lemma 1: Given an initial level $R_0$ and a final level $R_f$, such that $R_f > R_0$, the optimal trajectory $n^*_t$ that connects these two levels in a minimum time satisfying the constraint (2) is given by

$$n^*_t = R_0 \exp\left(\frac{t}{T}\right) \text{ for } 0 \leq t \leq T \ln\left(\frac{R_1}{R_0}\right)$$

Proof: From (2) and the definition of instantaneous inverse period (1), we obtain

$$\frac{dn}{dt} \leq \frac{1}{T}$$

Next,

$$\frac{dn}{n} \leq \frac{1}{T} dt$$

Integrating the last inequality gives

$$\int_{n_0}^{n} \frac{1}{n} \, dn \leq \int_{0}^{t} \frac{1}{T} \, d\tau$$

$$\ln\left(\frac{n}{n_0}\right) \leq \frac{t}{T}$$

$$n \leq R_0 \exp\left(\frac{t}{T}\right)$$

Consequently, to attain $R_1$ in a minimum time, $n_t$ must be equal to its upper bound, that is, $R_0 \exp\left(\frac{t}{T}\right)$. Under this condition, when $n^*_t = R_1$, we obtain

$$\hat{t} = T \ln\left(\frac{R_1}{R_0}\right)$$

In general, the optimal trajectory can be expressed as

$$n^*_t = \begin{cases} R_0 \exp\left(\frac{t}{T}\right); & 0 \leq t \leq T \ln\left(\frac{R_1}{R_0}\right) \\ R_1; & t > T \ln\left(\frac{R_1}{R_0}\right) \end{cases}$$

Once we have calculated the optimal trajectory, another important question emerges: how to force the nuclear system to follow this trajectory. To solve this new problem, first of all, we need a representation of the nuclear dynamics. Although there exists a physical model for this purpose, this one presents some drawbacks as it is explained in the following section.
3. PHYSICAL MODELING OF A NUCLEAR REACTOR: MAIN DRAWBACKS

A rigorous modeling from first principles for a nuclear reactor requires the use of the neutron diffusion and transport theory. The so-obtained model consists of a system of partial differential equations which is too complicated to be of any practical utility. However, starting from certain assumptions such as the independence of the space effects and considering one unique mechanism of thermal feedback, the model based on partial differential equations can be simplified into the following eighth order nonlinear model [10] valid for small or medium size reactors such as the nuclear research reactors in which we are interested.

\[ \dot{n}_t = \frac{\alpha T_o - \beta}{\Lambda} n_t + \sum_{i=1}^{6} \lambda_i C_{i,t} - \frac{\alpha}{\Lambda} n_t T_t + \frac{1}{\Lambda} n_t v_t \]  
(4)

\[ \dot{C}_{i,t} = \frac{\beta_i}{\Lambda} n_t - \lambda_i C_{i,t}, \quad i = 1, \ldots, 6 \]  
(5)

\[ \dot{T}_t = K (n_t - n_o) - \gamma (T_t - T_o) \]  
(6)

where the constant parameters are defined as: \( \Lambda \) is the effective prompt neutron lifetime (s), \( \lambda_i \) is the radioactive decay constant of \( i \)-th group neutron precursors (s\(^{-1}\)), \( \beta_i \) is the fraction of \( i \)-th group delayed neutrons. \( \beta \) is the total delayed neutron fraction \( \beta = \sum_{i=1}^{6} \beta_i \), \( a \) is the negative temperature reactivity coefficient (\( {\text{C}^\circ}/\text{s} \)), \( K \) is the reciprocal of the reactor heat capacity (\( \text{C}^\circ/(\text{W} \cdot \text{s}) \)), \( \gamma \) is the reciprocal of mean time for heat transfer to the coolant (s\(^{-1}\)), \( T_o \) is the initial average temperature. \( n_0 \) is the initial power which is provided by an external source. Likewise, the model variables are defined as: \( n \) is the neutron power (W), \( C_i \) is the power of the \( i \)-th group delayed neutron precursors (W) – since there exist six groups of delayed neutron precursors, equation (5) in fact represents six differential equations of first order. \( T_t \) is the fuel average temperature (\( {\text{C}^\circ} \)), and \( v_t \) is the external reactivity (dimensionless). The external reactivity is associated with the displacement of the control rods. The relationship between these variables can be inferred as a static mapping by correlations determined empirically and off-line. Since it is possible to determine this relationship, from now on the external reactivity can be considered as the control input of the nuclear system. In particular, the nominal parameters, corresponding to a TRIGA MARK III-type nuclear research reactor located in Mexico [11] are as follows: \( \beta_i = 0.240 \times 10^{-3} \), \( \beta_2 = 1.410 \times 10^{-3} \), \( \beta_3 = 1.255 \times 10^{-3} \), \( \beta_4 = 2.525 \times 10^{-3} \), \( \beta_5 = 0.737 \times 10^{-3} \), \( \beta_6 = 0.266 \times 10^{-3} \), \( \beta = 6.433 \times 10^{-3} \), \( \lambda_1 = 0.0124 \text{s}^{-1} \), \( \lambda_2 = 0.0305 \text{s}^{-1} \), \( \lambda_3 = 0.1140 \text{s}^{-1} \), \( \lambda_4 = 0.3013 \text{s}^{-1} \), \( \lambda_5 = 1.1360 \text{s}^{-1} \), \( \lambda_6 = 3.0130 \text{s}^{-1} \), \( \alpha = 0.01359875 \text{C}^{-1} \), \( \Lambda = 38 \text{ps} \), \( \gamma = 0.25 \), \( K = 1/5.21045 \times 10^3 \text{C}^\circ/(\text{W} \cdot \text{s}) \) whereas the ranges for some variables of the same reactor operating on standard conditions are: \( n \) from 1W to 1.1MW, \( v_t \) from 0 to 1.4354. On the other hand, due to the assumptions employed to deduce it, discrepancies between the actual plant and the physical model here studied are inevitable. Next, some limitations and drawbacks associated with the use of this model are summarized: 1) Although the parameters were assumed to be constants, in fact, they vary according to changes in operating conditions; 2) The effect of sensors and actuators is not considered; 3) Due to instruments for measuring power of 6 groups of delayed neutron precursors in a nuclear reactor are not available, a control design based on this model could result in complex structured closed loop system since a robust nonlinear observer is required; 4) Xenon induced power oscillations are not considered. Besides, there is no any instrument for measuring xenon and iodine concentration in nuclear reactors. To overcome these drawbacks and obtain a convenient mathematical model of the
reactor in the following section we propose to approximate the measurable nuclear dynamics by means of a generalized Hopfield neural network. Before finishing this section, two facts should be observed: First, if the output of the system is \( n_i \), then the relative degree of the physical model is 1. Second, although we do not know the current values of the reactor's parameters we can assure that these ones are always positive. Consequently, the dynamics associated with the \( i \)th group delayed neutron precursor power as well as the fuel temperature is stable. This can be seen from the equations (5) and (6), respectively. At this point, it is important to remember that nuclear research reactors, in particular those based on the principle of the inherent safety, are stable systems and a tragic accident such as Chernobyl explosion could never occur in them. In fact, research reactors are systems so safe that many of them are located on the campuses of different universities around the world.

4. HOPFIELD NEURAL NETWORK APPLIED TO SYSTEM IDENTIFICATION

One of the most popular and simple models of artificial neural networks was proposed by Hopfield in [12]. Typically, this network has been used to solve associative memory, pattern recognition, and optimization problems. However, some works have been reported in which Hopfield network is utilized for system identification and control [13], [14] and [15]. The structure of a Hopfield neural network [16] is given by

\[
\frac{d}{dt} \hat{x}_t = A \hat{x}_t + W_{1,t} \sigma(\hat{x}_t) + v_t
\]

where \( \hat{x}_t \in \mathbb{R}^n \) is the state of the neural network. \( A := \text{diag}([a_1 \ldots a_n]) \) is a \( n \times n \) real diagonal matrix with \( a_i < 0, \alpha, \epsilon, \cdot \) is an activation vector-function with sigmoidal components, that is, \( \sigma(\cdot) := \{ \sigma_1(\cdot), \ldots, \alpha, \epsilon, \cdot \}^T \)

\[
\sigma_j(z) := \frac{a_j}{1 + \exp \left( -\sum_{j=1}^n b_{ij} \hat{x}_{j,t} \right)} \quad \text{for } j = 1, \ldots, n \text{ and any } z \in \mathbb{R}^n
\]

\( v_t \in \mathbb{R}^n \) is the control input, and \( W_{1,t} \in \mathbb{R}^{n \times n} \) is the weight matrix which is continuously modified by an appropriate learning law in order to Hopfield neural network be able to approximate the measurable dynamic behavior of an unknown system of the form

\[
\dot{x}_t = h(x_t) + v_t
\]

where \( h : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is an unknown vector-valued function. In order to be able to approximate a wider class of systems, Hopfield neural network should be generalized adding an input weight matrix \( W_{2,t} \in \mathbb{R}^{n \times n} \) such that

\[
\frac{d}{dt} \hat{x}_t = A \hat{x}_t + W_{1,t} \sigma(\hat{x}_t) + W_{2,t} \phi(\hat{x}_t) v_t
\]

where \( \phi(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n} \) is a diagonal matrix function with sigmoidal components given by
Using (7), now it is possible to approximate unknown systems of the form

\[ \dot{x}_t = h(x_t) + g(x_t)v_t \]

where \( g : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is an unknown vector-valued function.

### 4.1 Uncertain Dynamics and Basic Assumptions

Suppose that the unmeasurable dynamics of a nuclear research reactor is stable. Likewise, consider that the \( n \)-order uncertain measurable dynamics of this system can be described, in general, by

\[ \dot{x}_t = f(x_t, v_t, t) \]  

(8)

where \( x_t \in \mathbb{R}^n \) represents the measurable state of the system, \( v_t \in \mathbb{R}^m \) is the control input, and \( f : \mathbb{R}^{n+m+1} \rightarrow \mathbb{R}^n \) is an unknown vector-valued function. Notice that an alternative representation for (8) always could be done as follows:

\[ \dot{x}_t = A x_t + W^0_1 \sigma (x_t) + W^0_2 \phi (x_t) v_t + \Delta f(x_t, v_t, t) \]  

(9)

where \( A, v_t, \sigma (\cdot) \) and \( \phi (\cdot) \) are as in (7) but now \( W^0_1 \in \mathbb{R}^{n \times n} \) and \( W^0_2 \in \mathbb{R}^{n \times n} \) are constant matrices which can be found before the learning process based on available a priori data (the so-called training period), and \( \Delta f(x_t, v_t, t) \) is an error term which could comprehend both bounded perturbations and unmodeled dynamics.

Hereafter we consider that the following assumptions are complied:

**A.1)** System (8) satisfies the (uniform on \( t \)) Lipschitz condition, that is,

\[ \| f (x, u, t) - f (z, v, t) \| \leq L_1 \| x - z \| + L_2 \| u - v \| \]

\( x, z \in \mathbb{R}^n; \ u, v \in \mathbb{R}^m; \ 0 \leq L_1, L_2 < \infty \)  

(10)

**A.2)** The functions \( \sigma (\cdot) \) and \( \phi (\cdot) \) satisfy sector conditions:

\[ \tilde{\sigma}_t^T \Lambda_\sigma \tilde{\sigma}_t \leq \Delta_t^T D_\sigma \Delta_t \]

\[ \tilde{v}_t^T \tilde{\phi}_t^T \Lambda_\phi \tilde{\phi}_t \tilde{v}_t \leq \Delta_t^T D_\phi \Delta_t \| v_t \|^2 \]

where

\[ \Delta_t := \hat{x}_t - x_t \]  

(11)
\[ \tilde{\sigma}_t := \sigma(\tilde{x}_t) - \sigma(x_t) \]
\[ \tilde{\phi}_t := \phi(\tilde{x}_t) - \phi(x_t) \]

and \( \Lambda_\sigma \in \mathbb{R}^{n \times n} \), \( D_\sigma \in \mathbb{R}^{n \times n} \), \( \Lambda_\phi \in \mathbb{R}^{n \times n} \), \( D_\phi \in \mathbb{R}^{n \times n} \) are known constant positive definite matrices.

A.3) Admissible controls are bounded, to be precise.

\[ U^{adm} := \{ u_t : \| u_t \| \leq \tilde{v} < \infty \} \]

Besides, \( u_t \) is such that does not violate the existence of the solution to ODE (8).

A.4) The error term is bounded by

\[ \| \Delta f(x_t, u_t, t) \|_{\Lambda_f} \leq \tilde{\eta} \]

where \( \Lambda_f \in \mathbb{R}^{n \times n} \) is a constant positive definite matrix.

A.5) The matrices \( W_1^0 \) and \( W_2^0 \) are bounded in the following sense

\[ W_1^0 \Lambda_\sigma^{-1} (W_1^0)^T \leq \overline{W}_1 \]
\[ W_2^0 \Lambda_\phi^{-1} (W_2^0)^T \leq \overline{W}_2 \]

where \( \overline{W}_1 \in \mathbb{R}^{n \times n} \), \( \overline{W}_2 \in \mathbb{R}^{n \times n} \) are known positive definite matrices.

A.6) There exits a strictly positive definite matrix \( Q_0 \) such that if the matrices \( R \) and \( Q \) are defined as

\[ R := \overline{W}_1 + \overline{W}_2 + \Lambda_f^{-1} \]
\[ Q := D_\sigma + \overline{u} D_\phi + Q_0 \]

then the following matrix Riccati equation

\[ A^T P + PA + PRP + Q = 0 \]

has a positive solution \( P \) (in [17] there are given conditions for matrices \( A, R, \) and \( Q \) which guarantee the existence of \( P > 0 \)).

It is worth mentioning that the preceding assumptions are not unusual. On the contrary, they are generally met for physically meaningful dynamic systems and a nuclear reactor is not the exception.

### 4.2 New Differential Learning Law

In this paper, to adjust the weight matrices \( W_{1,t} \) and \( W_{2,t} \) of the generalized Hopfield neural network (7), we propose to utilize the following leaning law:
\[ \dot{W}_{1,t} = -K_1 P \Delta_t \sigma(\tilde{x}_t) - \frac{\lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right)}{2} \dot{W}_{1,t} \]
\[ \dot{W}_{2,t} = -K_2 P \Delta_t u_t^T \phi(\tilde{x}_t)^T - \frac{\lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right)}{2} \dot{W}_{2,t} \]  

(15)

where \( K_1 \) and \( K_2 \) are positive definite matrices which are selected by the designer, \( P \) is the solution of the matrix Riccati equation given by \( (14) \), and

\[ \dot{W}_{1,t} := W_{1,t} - W_{1}^0 \]
\[ \dot{W}_{2,t} := W_{2,t} - W_{2}^0 \]  

(16)

Next, the basic result about the identification process of measurable dynamics \( (8) \) by the neural network \( (7) \) is formulated:

**Theorem 1:** If the assumptions \( A.1-A.6 \) are satisfied and the weight matrices \( W_{1,t} \) and \( W_{2,t} \) of the generalized Hopfield neural network \( (7) \) are adjusted by the differential learning law \( (15) \) then

a) both the identification error and the weights are bounded:

\[ \Delta_t, W_{1,t}, W_{2,t} \in L_\infty \]  

(17)

b) the identification error has the following upper bound:

\[ \lim_{t \to \infty} \sup_{z \geq t} \Delta_t^T P \Delta_z \leq \frac{\eta}{\lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right)} \]

(18)

The proof of this theorem can be consulted in the appendix 1.

**Remark 1:** If we have no any error term \( (\Delta f(x_t, v_t, t) = 0) \), we obtain \( \eta = 0 \) and, hence, from (18) we can conclude that the asymptotic stability of the identification error is guaranteed, i.e.,

\[ \lim_{t \to \infty} \sup_{z \geq t} \Delta_t^T P \Delta_z = 0 \]

Now then, in accordance with section 3, the relative degree of the physical model is 1, and the dynamics associated with both the power of the \( t \)th group neutron precursors and the fuel temperature is stable. Taking into account these facts and since the final objective is to carry out the control of the neutron power uniquely, the identification of the nuclear system can be accomplished using only the external reactivity \( \nu_t \) and the neutron power \( n_t \). Consequently, under these conditions, \( x_t \in \mathcal{R} \) and both generalized Hopfield network \( (7) \) and the learning law \( (15) \) become simple first order differential equations. Likewise, the matrix Riccati equation \( (14) \) is now a second order algebraic equation.

### 5. Tracking Control Based on Hopfield Neural Network

In this section, on the basis of the generalized Hopfield neural network \( (7) \), a strategy of indirect adaptive control (see Figure 2) is presented to accomplish the tracking of a given reference trajectory.
Theorem 2: If the state $x_t \in \mathbb{R}$ of the nonlinear system (8) is forced to track a reference trajectory $x_t^* \in \mathbb{R}$ by the following control law

$$v_t = \frac{\dot{x}_t^* - Ax_t - W_2c(x_t) - c(x_t - x_t^*)}{W_2c(x_t)}$$

(19)

where $c$ is a positive constant which is selected by the designer such that $c > 1/2$, then the tracking error which is defined as $e_t := x_t - x_t^*$ has the following upper bound:

$$\|e_t\| \leq \sqrt{\|e_0\|^2 + \frac{\eta}{2c - 1}}$$

Remark 2: In order to avoid the division by zero in (19), the learning law (15) should be constrained such that $W_2 > 0$, $\forall t \in \mathbb{R}^-$.  

Figure 2. Blockdiagram of proposed control system

6. SIMULATION RESULTS

In this section, the identification and control process proposed in this work is illustrated by numeric simulation. In the first place, the optimal trajectory is calculated in accordance with (3). To study the control system performance, a very large change of the neutron power is considered selecting the initial and final levels as $R_0 = 1W$ and $R_1 = 1,000,000W$, respectively. The nominal period is selected as $T = 3s$. Thus, the minimum transient time to switch between the power levels $R_0$ and $R_1$ is equal to $41.45s$. Due to the ample variation of $n$, this variable is normalized, that is, divided by its corresponding maximum value which is denoted by $g$ ($g$ is considered here equal to $1,100,000W$). Thus,
where \( x_r \) represents the state of the measurable unknown dynamics (8). Such normalization does not affect the identification result. Instead, it permits to the generalized Hopfield network (7) works satisfactorily. Due to the normalization process of \( n_i \), the optimal trajectory (3) must also be normalized. Thus,

\[
x^*_i = \frac{n^*_i}{g}
\]

During the simulation, instead of the real plant, the eighth order nonlinear model (equations (4), (5), and (6)) is used with the nominal parameters presented in section II. The initial power \( n_0 \) is selected equal to 0.9W whereas the initial value for the estimated power \( n_0 \) is selected equal to 1W. By hy to test, the parameters of the generalized Hopfield neural network (7) and the learning law (15) are selected as follows: \( A = -0.5 \), \( R = 0.8533 \), \( Q_0 = 1 \), \( Q = 3 \), \( P = 15 \), \( K_1 = 4780 \), \( K_2 = 100 \), \( W_{1,0} = 0.1 \), \( W_{1,0} = 0.01 \), \( W_{2,0} = 0.8 \) and \( W_{2,0} = 0.5 \), where \( W_{1,0} \) and \( W_{2,0} \) represent the initial values when \( i = 0 \) for the weight matrices \( W_{1,i} \) and \( W_{2,i} \). With respect to the parameter \( W_{1,0} \), although this one could be determined by a procedure based on least squares, the so obtained value could generate a very big error of identification at the beginning of the process. Thereby, we have preferred to employ small values for both \( W_{1,0} \) and \( W_{2,0} \). On the other hand, the parameter \( c \) of the control law (19) that obligates to the physical model to follow the optimal trajectory (3) is selected equal to 40. The results of the control process are displayed in Figures 3-6. In Figure 3.a, no difference can be appreciated between the optimal trajectory \( n^*_i \) (continuous line) and the power \( n_i \) of the physical model (dashed line). Thereby, in Figure 3.b, a close-up to the Figure 3.a is achieved. Now, the difference between the initial conditions is evident.

![Figure 3](image_url)

Figure 3. a) Control process of the power \( n_i \) corresponding to physical model; b) Close-up of a) in which the difference between the initial conditions is evident.
To quantify the performance of the controller and due to the ample range of values associated to the neutron power (6 decades), we use the following relative percentage error of tracking defined as

$$\tilde{e}_t = \frac{n_t - n_t^*}{n_t^*} \times 100\%$$

In accordance with Figure 4, the maximum of this tracking error is equal to 10% and occurs at the beginning of the control process. However, after only 2.5 seconds, this error is always less than 0.165%.

![Figure 4](image)

Figure 4. a) Relative percentage error $\tilde{e}_t$ for tracking process; b) Close-up of a)

In Figure 5, the control signal is showed. This signal, in general, is sufficiently smooth. Finally, from Figure 6, it can be verified that the constraint of the inverse period (2) is always satisfied, that is, the inverse period is always less than or equal to 1/3 when $T = 3$. To test the robustness of the proposed strategy, the following numeric experiment is achieved: whereas the parameters of the controller-identifier stay without any modification, the following new conditions are applied to the physical model 1) the parameters of this model are changed in accordance with the specifications of the table 1 (in some cases, these changes are greater than 200%), 2) to model some effects associated with real instrumentation, the output $n_t$ of the physical model is corrupted by a white noise which magnitude is always equal to 1% with independence of the power level, and 3) once the level $R_1$ is reached, to simulate the Xenon feedback effects, a sine function with amplitude equal to 0.1 and frequency equal to 0.01 Hz is added to the external reactivity $n_t$ (naturally, the real cycle of Xenon has a much smaller frequency). The results obtained under these new conditions are showed in Figure 7-8 for the first 100 seconds of the simulation. In accordance with Figure 7, the relative percentage error of tracking shows an acceptable quality. Lastly, as it can be appreciated from Figure 8, in spite of the extreme conditions, the constraint of the inverse period is still satisfied.

7. CONCLUSIONS

In this work, the application of an indirect adaptive controller for nuclear research reactors based on a generalized Hopfield neural network has been presented. The objective is to
accomplish the switching between different levels of power in a minimum time satisfying the inverse period constraint. Since no observer is required, the proposed controller has a simple structure. Besides, the tuning of the identifier-controller parameters can be accomplished with relative quickness and easiness. It is important to point out that the proposed strategy does not need any knowledge about the parameters of the physical model to control the reactor. To verify its effectiveness, the controller was tested by numeric simulation with the physical model used only as a data generator instead of the real plant. The simulation results confirm the workability and robustness of the suggested approach. Since nowadays the control system of many research reactors includes a digital computer, the implementation of the proposed strategy should not be a complex problem. Naturally, in order to accomplish real tests, it is advisable first to calculate the optimal trajectory with large values for the period $T$ and, once satisfactory results have been obtained, successively the magnitude of the period should be decreased to values next to the nominal period of each particular reactor.
Table I. New Values for the Parameters of the Reactor Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>New value</th>
<th>Percentage Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.14e-3</td>
<td>-41.67%</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.098e-3</td>
<td>-22.13%</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.972e-3</td>
<td>-22.54%</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>1.972e-3</td>
<td>-21.90%</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>0.717e-3</td>
<td>-2.71%</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>0.239e-3</td>
<td>-10.15%</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>5.138e-3</td>
<td>-20.13%</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.0126</td>
<td>1.61%</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.0306</td>
<td>0.33%</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0.119</td>
<td>4.39%</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>0.318</td>
<td>5.54%</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>1.19</td>
<td>4.75%</td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td>3.11</td>
<td>3.22%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.035</td>
<td>157.38%</td>
</tr>
<tr>
<td>$K$</td>
<td>6.3334e-005</td>
<td>230%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.63</td>
<td>215%</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1.3e-6</td>
<td>-96.558%</td>
</tr>
</tbody>
</table>

Figure 7, a) Relative percentage error $\tilde{e}_t$ for tracking process when new conditions for the physical model are applied; b) Close-up of a).
Figure 8. Instantaneous inverse period when new conditions for the physical model are applied.

REFERENCES

APPENDIX I

Proof of theorem 1: Before beginning the analysis, the dynamics of the identification error must be determined. From (11), the first derivative of $A$, is

$$\dot{\Delta}_t = \dot{\hat{x}}_t - \dot{x}$$  \hspace{1cm} (20)

Substituting (7) and (9) into (20) yields

$$\Delta_t = AA_t + W_{1,t}\sigma(\hat{x}_t) - W_{1,t}^0\sigma(x_t) + W_{2,t}\phi(\hat{x}_t)v_t - W_{2,t}^0\phi(x_t)v_t - Af$$  \hspace{1cm} (21)

Adding and subtracting the terms $W_{1,t}^0\sigma(\hat{x}_t)$ and $W_{2,t}^0\phi(\hat{x}_t)v_t$ and taking into account equations (12) and (16), (21) can be expressed as

$$\dot{\Delta}_t = A\Delta_t + \widetilde{W}_{1,t}\sigma(\hat{x}_t) + W_{1,t}^0\tilde{\sigma}_t + \widetilde{W}_{2,t}\phi(\hat{x}_t)v_t + W_{2,t}^0\tilde{\phi}_t v_t - \Delta f$$  \hspace{1cm} (22)

To begin the analysis, we select the following non-negative function

$$V_t := \Delta_t^T P \Delta_t + \text{tr}[\widetilde{W}_{1,t}^TK_1^{-1}\widetilde{W}_{1,t}] + \text{tr}[\widetilde{W}_{2,t}^TK_2^{-1}\widetilde{W}_{2,t}]$$  \hspace{1cm} (23)

where $P$ is the positive solution for the matrix Riccati equation given by (14). The first derivative of $V_t$ is

$$\dot{V}_t = \frac{d}{dt}(\Delta_t^T P \Delta_t) + \frac{d}{dt}\text{tr}[\widetilde{W}_{1,t}^TK_1^{-1}\widetilde{W}_{1,t}] + \frac{d}{dt}\text{tr}[\widetilde{W}_{2,t}^TK_2^{-1}\widetilde{W}_{2,t}]$$  \hspace{1cm} (24)
\[
\frac{d}{dt} \text{tr} \left[ \tilde{W}_{2,t}^T K_2^{-1} \tilde{W}_{2,t} \right] = -2 \Delta_t^T P \tilde{W}_{2,t} \phi(\tilde{x}_t) v_t - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \tilde{W}_{2,t}^T K_2^{-1} \tilde{W}_{2,t} \right] \quad (29)
\]

Finally, substituting (26), (28), and (29) into equation (24), \( \dot{V}_t \) can be expressed as

\[
\dot{V}_t = 2 \Delta_t^T P A \Delta_t + 2 \Delta_t^T P W_1^0 \tilde{\sigma}_t + 2 \Delta_t^T P W_2^0 \dot{\phi}_t v_t - 2 \Delta_t^T P \Delta tf
\]

\[
-\lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \tilde{W}_{1,t}^T K_1^{-1} \tilde{W}_{1,t} \right] - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \tilde{W}_{2,t}^T K_2^{-1} \tilde{W}_{2,t} \right] \quad (30)
\]

Now, we need to find an upper bound for \( \dot{V}_t \). To accomplish this task, first we consider the term \( 2 \Delta_t^T P W_1^0 \tilde{\sigma}_t \). Since this term is a scalar, it is possible to express it alternatively as

\[
2 \Delta_t^T P W_1^0 \tilde{\sigma}_t = \Delta_t^T P W_1^0 \tilde{\sigma}_t + \tilde{\sigma}_t^T \left( W_1^0 \right)^T P \Delta_t
\]

using the matrix inequality proven in [17]

\[
X^T Y + Y^T X \leq X^T \Gamma^{-1} X + Y^T \Gamma Y \quad (31)
\]

which is valid for any \( X, Y \in \mathbb{R}^{n \times k} \) and for any positive definite matrix

\[
0 < \Gamma = \Gamma^T \in \mathbb{R}^{k \times k}. \quad 2 \Delta_t^T P W_1^0 \tilde{\sigma}_t \text{ can be bounded by }
\]

\[
2 \Delta_t^T P W_1^0 \tilde{\sigma}_t \leq \Delta_t^T P W_1^0 \Lambda \Delta_t + \tilde{\sigma}_t^T \Lambda \tilde{\sigma}_t
\]

but, from the assumptions \textbf{A.2} and \textbf{A.5}, we can conclude

\[
2 \Delta_t^T P W_1^0 \tilde{\sigma}_t \leq \Delta_t^T P \tilde{W}_1 P \Delta_t + \Delta_t^T \tilde{D}_\phi \Delta_t \quad (32)
\]

likewise, using the inequality (31) into \( 2 \Delta_t^T P W_2^0 \dot{\phi}_t v_t \), we have

\[
2 \Delta_t^T P W_2^0 \dot{\phi}_t v_t = \Delta_t^T P W_2^0 \dot{\phi}_t v_t + u_t^T \dot{\phi}_t^T \left( W_2^0 \right)^T P \Delta_t
\]

\[
\leq \Delta_t^T P W_2^0 \Lambda^{-1} \left( W_2^0 \right)^T P \Delta_t + u_t^T \dot{\phi}_t^T \Lambda \dot{\phi}_t u_t
\]

\[
\leq \Delta_t^T P \tilde{W}_2 P \Delta_t + \tilde{u}_t^T \tilde{D}_\phi \tilde{u}_t \quad (33)
\]

This last inequality is concluded by the assumptions \textbf{A.2}, \textbf{A.3}, and \textbf{A.5}. On the other hand, the following inequality is a corollary from (31):

\[
-Z^T Y - Y^T Z \leq Z^T \Gamma^{-1} Z + Y^T \Gamma Y
\]

which is valid for any \( Z, Y \in \mathbb{R}^{n \times k} \) and for any positive definite matrix

\[
0 < \Gamma = \Gamma^T \in \mathbb{R}^{k \times k}. \text{ Using this result to bound the term } -2 \Delta_t^T P \Delta f, \text{ we find that }
\]
Each term of (24) will be calculated separately. For \( \frac{d}{dt} (\Delta_t^T P \Delta_t) \), we have

\[
\frac{d}{dt} \Delta_t^T P \Delta_t = 2\Delta_t^T P \dot{\Delta}_t
\]  

(25)

substituting (22) into (25) yields

\[
\frac{d}{dt} \Delta_t^T P \Delta_t = 2\Delta_t^T P \Delta_t + 2\Delta_t^T P \tilde{W}_{1,t} \sigma(\tilde{x}_t) + 2\Delta_t^T P \tilde{W}_{1,t} \delta(\tilde{x}_t) \nu_t + 2\Delta_t^T P \tilde{W}_{2,t} \delta(\tilde{x}_t) \nu_t + 2\Delta_t^T P W_{1,t} \phi_t \nu_t - 2\Delta_t^T P \Delta_f
\]  

(26)

On the other hand, for \( \frac{d}{dt} \text{tr} [\tilde{W}_{1,t} K_1^{-1} \tilde{W}_{1,t}] \), using several properties of the trace of a matrix, we obtain

\[
\frac{d}{dt} \text{tr} [\tilde{W}_{1,t} K_1^{-1} \tilde{W}_{1,t}] = \text{tr} \left[ \frac{d}{dt} (\tilde{W}_{1,t} K_1^{-1} \tilde{W}_{1,t}) \right]
\]

\[
= \text{tr} \left[ \dot{\tilde{W}}_{1,t} K_1^{-1} \tilde{W}_{1,t} + \tilde{W}_{1,t} K_1^{-1} \dot{\tilde{W}}_{1,t} \right]
\]

\[
= \text{tr} \left[ \dot{\tilde{W}}_{1,t} K_1^{-1} \tilde{W}_{1,t} \right] + \text{tr} \left[ \tilde{W}_{1,t} K_1^{-1} \dot{\tilde{W}}_{1,t} \right]
\]

\[
= 2\text{tr} \left[ \dot{\tilde{W}}_{1,t} K_1^{-1} \tilde{W}_{1,t} \right]
\]  

(27)

if (16) is differentiated then

\[
\dot{\tilde{W}}_{1,t} = \tilde{W}_{1,t}
\]

but \( \dot{\tilde{W}}_{1,t} \) is given by the learning law (15). Thus, substituting (15) into the last term of (27),

\[
\frac{d}{dt} \text{tr} [\tilde{W}_{1,t} K_1^{-1} \tilde{W}_{1,t}]
\]

can be calculated as

\[
\frac{d}{dt} \text{tr} [\tilde{W}_{1,t} K_1^{-1} \tilde{W}_{1,t}] = -2\text{tr} [\sigma(\tilde{x}_t) \Delta_t P K_1 \tilde{W}_{1,t}] - \lambda_{\min} (P^{-1}Q_0 P^{-1}) \text{tr} [\tilde{W}_{1,t} K_1^{-1} \tilde{W}_{1,t}]
\]

\[
= -2\Delta_t^T P \tilde{W}_{1,t} \sigma(\tilde{x}_t) - \lambda_{\min} (P^{-1}Q_0 P^{-1}) \text{tr} [\tilde{W}_{1,t} K_1^{-1} \tilde{W}_{1,t}]
\]  

(28)

Proceeding in a similar way for \( \text{tr} \left[ \tilde{W}_{2,t} K_2^{-1} \tilde{W}_{2,t} \right] \).
\[ -2 \Delta_t^T P \Delta_t \leq \Delta_t^T P \Lambda_f^{-1} P \Delta_t + \Delta_t^T \Lambda_f \Delta_t \]

but, in accordance with the assumption A.4

\[ -2 \Delta_t^T P \Delta_t \leq \Delta_t^T P \Lambda_f^{-1} P \Delta_t + \overline{\eta} \tag{34} \]

Substituting (32), (33), (34) into (30), the following bound for \( \dot{V}_t \) can be determined

\[ \dot{V}_t \leq 2 \Delta_t^T P A \Delta_t + \Delta_t^T P \overline{W}_1 P \Delta_t + \Delta_t^T P \overline{W}_2 P \Delta_t + \overline{u} \Delta_t^T D_o \Delta_t \]

\[ + \Delta_t^T \Lambda_f^{-1} P \Delta_t + \overline{\eta} - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \overline{W}_{1,t} K_1^{-1} \overline{W}_{1,t}^T \right] \]

\[ - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \overline{W}_{2,t} K_2^{-1} \overline{W}_{2,t}^T \right] \]

Adding and subtracting \( \Delta_t^T Q_0 \Delta_t \) into the right-hand side of the last inequality, the expression \( A^T P + PA + P (\overline{W} + \Lambda_f^{-1}) \) \( P + D_o + Q_0 \) is formed. However, this expression in accordance with the assumption A.6 is equal to zero. Then

\[ \dot{V}_t \leq - \Delta_t^T Q_0 \Delta_t + \overline{\eta} - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \overline{W}_{1,t} K_1^{-1} \overline{W}_{1,t}^T \right] \]

\[ - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \overline{W}_{2,t} K_2^{-1} \overline{W}_{2,t}^T \right] \]

Now, considering that

\[ \Delta_t^T Q_0 \Delta_t = \Delta_t^T P A \left( P^{-1} Q_0 P^{-1} \right) P^T \Delta_t \]

and using Rayleigh inequality [18], we obtain

\[ \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \Delta_t^T PA, \leq \Delta_t^T Q_0 \Delta_t \]

or else

\[ - \Delta_t^T Q_0 \Delta_t \leq - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \Delta_t^T P \Delta_t \]

consequently,

\[ \dot{V}_t \leq - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \Delta_t^T P \Delta_t - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \overline{W}_{1,t} K_1^{-1} \overline{W}_{1,t}^T \right] \]

\[ - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) \text{tr} \left[ \overline{W}_{2,t} K_2^{-1} \overline{W}_{2,t}^T \right] + \overline{\eta} \]

from the definition (23) for the non-negative function \( V_t \), finally \( \dot{V}_t \) can be bounded as

\[ \dot{V}_t \leq - \lambda_{\min} \left( P^{-1} Q_0 P^{-1} \right) V_t + \overline{\eta} \]
which implies that

\[ V_t \leq V_0 \exp(-\xi t) + \frac{\bar{\eta}}{\xi}(1 - \exp(-\xi t)) \]  

(35)

where \( \xi = \lambda_{\min}\left(P^{-1}Q_0P^{-1}\right) \). Since \( P \) and \( Q_0 \) are both positive definite matrices then \( \lambda_{\min}\left(P^{-1}Q_0P^{-1}\right) \) is always a positive scalar and therefore \( V_t \) is an upperly bounded function. But, by definition, \( V_t \) is also a non-negative function. Consequently, \( \Delta_t, W_{1,t}, W_{2,t} \in L_\infty \) and the first part of the theorem 1 has been proven. With respect to the last part of the theorem, from the definition (23), it is evident that \( \Delta_t^T P \Delta_t \leq V_t \) but, in accordance with (35), \( V_t \) is bounded by \( V_0 \exp(-\xi t) + \frac{\bar{\eta}}{\xi}(1 - \exp(-\xi t)) \). This means that

\[ \Delta_t^T P \Delta_t \leq V_0 \exp(-\xi t) + \frac{\bar{\eta}}{\xi}(1 - \exp(-\xi t)) \]

Finally, taking \( \lim \sup_{t \to \infty} \) when \( t \to \infty \), the last part of the theorem 1 has been proven. ■

APPENDIX II

Proof of theorem 2: We can see that another alternative representation for (8) is given by

\[ \dot{x}_t = Ax_t + W_{1,t}\sigma(x_t) + W_{2,t}\phi(x_t)u_t + \delta_t \]  

(36)

where \( \delta_t \) is a neural model error term. Since \( Ax_t + W_{1,t}\sigma(x_t) + W_{2,t}\phi(x_t)u_t \) with \( W_{1,t} \) and \( W_{2,t} \) adjusted by (15) is a better approximator of (8) than simply \( Ax_t + W_{1,0}\sigma(x_t) + W_{2,0}\phi(x_t)u_t \) then

\[ \|\delta_t\|_{\Lambda_j}^2 \leq \|\Delta f(x_t, u_t, \bar{\eta})\|_{\Lambda_j}^2 \]

However, from the assumption A.4, we have

\[ \|\delta_t\|_{\Lambda_j}^2 \leq \bar{\eta} \]

On the other hand, substituting the control law (19) into (36) and after some operations, yields

\[ \dot{x}_t = \dot{x}_t^* - c(x_t - x_t^*) + \delta_t \]  

(37)

considering that in accordance with the definition \( e_t := \dot{x}_t - x_t^* \) and consequently
\[ \dot{e}_t = \dot{x}_t - \dot{x}_t^* \] we can obtain from (37) that
\[ \dot{e}_t = -ce_t + \delta_t \] (38)
which is the dynamics of the tracking error. To analyze the behavior of this dynamics, we use the following Lyapunov function candidate
\[ V_t = e_t^2 \]
The first derivative of \( V_t \) is
\[ \dot{V}_t = 2e_t \dot{e}_t \] (39)
substituting (38) into (39) yields
\[ \dot{V}_t = -2ce_t^2 + 2e_t \delta_t \]
using the following inequality
\[ 2xy \leq x^2 + y^2 \]
which is valid for any \( x, y \in \mathbb{R} \), \( \dot{V}_t \) can be bounded as
\[ \dot{V}_t \leq -(2c - 1)V_t + \bar{\eta} \]
which implies that
\[ V_t \leq V_0 \exp(-\{(2c - 1)t\}) + \frac{\bar{\eta}}{2c - 1}(1 - \exp(-\{(2c - 1)t\})) \]
Since, by hypothesis, \( c > \frac{1}{2} \), then \( V_t \) can be bounded by
\[ V_t \leq V_0 + \frac{\bar{\eta}}{2c - 1} \]
Finally, we can conclude that
\[ \|e_t\| \leq \sqrt{V_0 + \frac{\bar{\eta}}{2c - 1}} \]

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