Abstract—The unified descriptive experiment design regularization (DEDR) method from a companion paper provides a rigorous theoretical formalism for robust estimation of the power spatial spectrum pattern of the wavefield scattered from an extended scene observed in the uncertain remote sensing (RS) environment. For the considered here imaging synthetic aperture radar (SAR) application, the proposed DEDR approach is aimed at performing, in a single optimized processing, SAR focusing, speckle reduction, and RS scene image enhancement and accounts for the possible presence of uncertain trajectory deviations. Being nonlinear and solution dependent, the optimal DEDR estimator requires rather complex signal processing operations ruled by the fixed-point iterative implementation process. To simplify further the processing, in this paper, we propose to incorporate the descriptive regularization via constructing the projections onto convex sets that enable us to factorize and parallelize the reconstructive image processing over the range and azimuth coordinates, design a family of such regularized easy-to-implement iterative algorithms, and provide the relevant computational recipes for their application to fractional imaging SAR. We also comment on the adaptive adjustment of the DEDR operational parameters directly from the actual speckle-corrupted scene images and demonstrate the effectiveness of the proposed adaptive DEDR techniques.

Index Terms—Convex solution sets, descriptive experiment design, de-speckling, regularization, spatial spectrum pattern (SSP), synthetic aperture radar (SAR).

List of Acronyms

AF Ambiguity function.
ASF Adaptive spatial filter.
DEDR Descriptive experiment design regularization.
ML Maximum likelihood.
MSF Matched spatial filter.
POCS Projections onto convex sets.
RASF Robust ASF.
RS Remote sensing.
SAR Synthetic aperture radar.
SFO Signal formation operator.
SSP Spatial spectrum pattern.


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I. INTRODUCTION

In a previous paper of this series [1], we have developed a general theory of the unified DEDR method for reconstructing the power SSP of the field scattered from an extended probing surface given a finite set of radar/SAR data recordings in the uncertain RS environment. The operational uncertainties are associated with the unknown statistics of random perturbations of the SFO in the turbulent medium, imperfect array calibration, finite dimensionality of measurements, multiplicative signal-dependent speckle noise, uncontrolled antenna vibrations, and random carrier trajectory deviations in the case of SAR. The general DEDR method has been constructed in [1] as an extension of the statistically optimal ML technique, in which the SSP estimation error is minimized in a descriptively balanced fashion (weighted maximization of spatial resolution over the minimization of the resulting noise energy) algorithmically coupled with the worst case statistical performance optimization-based convex regularization. We found the general-form DEDR-optimal solution to the inverse SSP reconstruction problem for the following two typical technically motivated operational scenarios: 1) single processed uncertain observation data realization and 2) ensemble of multiple independent processed uncertain data observations. The second scenario perfectly corresponds to the array radar applications [2]–[4], whereas the first one exactly matches the single-look imaging SAR systems [5]–[7]. Being nonlinear and solution dependent, the optimal general-form DEDR estimator constructed in [1] requires complex adaptive signal processing operations that involve also the proper construction of the regularizing POCS ruled by the adopted fixed-point contractive iteration process. For the real-world large-scale scenes, such general-form optimal DEDR method turns out to be computationally extremely consuming; therefore, it cannot be recommended as a procedure that is executable in (near) real computational time. (see [1, Sec. VI]).

In this paper, we design a family of computationally efficient techniques employing the idea of range–azimuth factorization with proper sparse robustification of the corresponding DEDR-related solution operators (SOs). Algorithmically, this is implemented via performing the POCS-based regularization adapted, in particular, to imaging SAR systems with fractionally synthesized aperture. We show how the iterative POCS regularization schemes lead to different easy-to-implement algorithms with
drastically reduced computational load, provide the relevant numerical recipes, and discuss their performances in comparison with the conventional SAR focusing techniques aggregated with the celebrated Lee adaptive despeckling filters based on the local statistics method [7]. Pursuing the DEDR approach to enhance the RS imagery, in this paper, we do not consider the alternative multilook SAR processing techniques to speckle reduction by averaging several frames obtained from a portion of the available azimuth spectral band [2], [7], as all those are performed at the expense of azimuth resolution. The crucial practical issue relates to the proper adjustment of the regularization parameters in the corresponding DEDR reconstructive operators to the particular uncertain RS operational scenario (recall that the regularization parameters balance the attained spatial resolution with the composite noise reduction [1]).

We propose to perform the adaptive adjustment of such regularization parameters using the empirical estimate of the multiplicative noise variance evaluated directly from the actual speckle-corrupted image of the remotely sensed scene. This alleviates the problem ill posedness related to the uncertainties in assigning the DEDR degrees of freedom [1] and leads to the new unified technique that aggregates the high-resolution RASF algorithm from the DEDR family [1] with the Lee adaptive despeckling filter [7]. Such an approach enables us to put, in a single DEDR-optimization frame, adaptive SAR focusing, speckle reduction, and RS scene image enhancement in the uncertain RS environment. The advantage in using the developed techniques over the previously proposed RS imaging and despeckling methods is demonstrated through the reported computer simulation experiments.

This paper is organized as follows. In Section II, we recall the main results of the DEDR method referring to the first paper of this series [1]. In Section III, we develop the POCS-regularized DEDR-related algorithms for enhanced SSP reconstruction with uncertain SAR observation data. The adaptive extension of the DEDR method is proposed in Section IV. Performance issues are featured in Section V followed by the simulation results with the relevant discussions in Section VI and concluding remarks in Section VII.

II. SUMMARY OF THE DEDR METHOD

A. Problem Formalism

Referring to the first paper of this series [1], the random signal \( u \) at the output of the SAR antenna moved by the carrier along the deviated linear trajectory \( \rho(t) \) in the time instance \( t \) relates to the field \( e \) scattered from the probing surface through the integral equation of observation [1]

\[
u(p) = \langle \tilde{S}(e) \rangle(p) + n(p) = \int \tilde{S}(p,r)e(r)dr + n(p)
\]  

(1)

where \( p = (t, \rho(t)) \) defines the time-space trajectory points [2]; the complex scattering function \( e(r) \) represents the random scene reflectivity over the probing surface in the plane of the scanned scene; \( r \) is a vector of the scan parameters, usually the polar, cylindrical, or Cartesian coordinates of the probing surface; and the uncertain SFO \( \tilde{S} \) is defined by the integral at the right hand in (1) with the nominal kernel \( S(p, r) = \langle \tilde{S}(p, r) \rangle \) specified by the time-space modulation of the signals employed in a particular imaging SAR system [2], [4]. The variations about the mean \( \delta S(p, r) = \tilde{S}(p, r) - S(p, r) \) pertain to the random perturbation component in the SFO. The employed symbols and uncertain SAR imaging problem geometry are illustrated in Fig. 1. For the detailed description of the uncertain data model, the reader can refer to the companion paper [1].

The SSP \( h(r) = (|e(r)|^2) \) represents the ensemble average of the squared modulus of the random complex reflectivity \( e(r) \) as a function over the analysis domain \( R \ni r \). The vector-form model of (1) is given by [1, eq. (18)] that we rewrite here as

\[
u = \tilde{S}e + n = Se + \Delta e + n
\]  

(2)

where \( u, n, \) and \( e \) define the vectors that are composed of the coefficients \( \{u_m\}, \{n_m\}, \) and \( \{e_k\} \) of the discrete-form approximations of the fields \( u, n, \) and \( e \) with respect to the selected orthogonal decomposition function set \( \{h_m(p)\} \) in the observation domain and the pixel set \( \{g_k(r)\} \) in the scene domain, respectively [1]. The matrix-form representation of the uncertain SFO in (2) is given by [1, eq. (16)], i.e.,

\[
\tilde{S} = S + \Delta.
\]  

(3)

The \( M \times K \) nominal SFO matrix \( S \) in (2) and (3) is composed of the scalar products \( \{[Sg_k, h_m](u)\} \) [1], while all problem model uncertainties are attributed to the distortion term \( \Delta \). We refer to the first paper of this series, where the distortions in the random medium were explained based on the propagation theory models [10]–[12]. Note that, in practice, one cannot attribute the exact portion of the composite SFO perturbation term \( \Delta \) to a particular source of disturbances, thus cannot separate in (3) the uncertainties caused by the turbulent medium effects, speckle noise, or the observation mismatch errors as those are randomly mixed in \( \Delta \). Moreover, the statistical characteristics of such composite random perturbation matrix \( \Delta \) are unknown to the observer [see [1, Sec. II] for more details on the uncertain model (3)]. These practical aspects motivated our adopting in [1] the robust statistical treatment of the irregular SFO perturbations \( \Delta \) as a random zero-mean matrix with the bounded second-order moment, i.e.,

\[
\langle \Delta \rangle = 0, \quad \langle ||\Delta||^2 \rangle = \langle tr\{\Delta^{\dagger}\Delta\} \rangle \leq \eta
\]

(4)
where \(|\Delta|^2 = \text{tr}\{\Delta\Delta^+\}\) defines the squared matrix norm, \(\text{tr}\{\cdot\}\) is the trace operator, superscript \(^+\) defines the Hermitian conjugate (conjugate transpose), and \(\eta\) is the bounding constant [1].

Because of the incoherent nature of the scattering function \(e(r)\), vector \(e\) in the equation of observation (2) is characterized by a diagonal correlation matrix \(R_e = \text{diag}(b) = D(b)\) in which the \(K\) vector \(b\) of the principal diagonal (composed of the elements \(b_k = |e_k|^2; k = 1, \ldots, K\)) is referred to as the vector-form SSP. The problem that we solved in the first paper of this series [1] was to derive an estimate \(\hat{b}\) of the SSP vector and to reconstruct the desired SSP distribution

\[
\hat{b}_{(k)}(r) = \sum_{k=1}^{K} \hat{b}_k r_{k}(r)
\]

over the pixel-formatted observation scene \(R \supset r\) by processing the data vector \(u\) (in the operational scenario with the single processed uncertain data realization) or \(J > 1\) whatever the available recorded independent realizations \(\{u_{ij}; j = 1, \ldots, J\}\) of the data (in the scenario with multiple observations) collected with a particular system operating in the uncertain RS environment.

### B. DEDR Estimators

To alleviate the ill posedness of the SSP reconstruction problem (5) with the uncertain observation model (2)–(4), the DEDR method was constructed in [1] given by [1, eq. (44)]

\[
\hat{b}_{\text{DEDR}} = \text{FyF}^+ (Y\Sigma^1\text{Y})^3\text{SK}_\text{diag} (6)
\]

that reconstructs the SSP vector \(\hat{b}\) via applying the DEDR-optimal SO

\[
F = K\Sigma^1 R_{\Sigma^{-1}}^1
\]

to the data matrix \(Y\) that is composed of uncertain data measurements [1, eqs. (28) and (30)], i.e., the rank-1 (ill-conditioned) outer product matrix

\[
Y = Y_{(\text{rank}-1)} = uu^+
\]

in the scenario with single recorded data realization (e.g., single-look imaging SAR applications) and the rank-\(J\) empirical estimated correlation matrix

\[
Y = Y_{(\text{rank}-J)} = (1/J) \sum_{j=1}^{J} u_{(j)}u_{(j)}^+
\]

in the scenario with \(J > 1\) independent multiple observations.

\(S^+\) in the SO (7) represents the adjoint (Hermitian conjugate [1]) to the nominal SFO matrix \(S\), and \(R_{\Sigma^{-1}}\) is the inverse of the augmented (diagonal loaded) noise correlation matrix defined by [1, eq. (41)], \(R_S = R_{\Sigma^{-1}}(\beta) = (R_n + \beta I)\). In the practical RS scenarios [3], [5], [7], [13] (and, specifically, in the SAR imaging applications [1], [4], [14], [15]), it is a common practice to accept the robust white additive noise model, i.e., \(R_n = N_0 I\), attributing the unknown correlated noise component as well as the speckle to the composite uncertain noise term \(\Delta e\) in (2), in which case

\[
R_S = N_\Sigma I \quad N_\Sigma = N_0 + \beta
\]

with the composite noise variance \(N_\Sigma = N_0 + \beta\), the initial \(N_0\) augmented by the loading factor \(\beta = \gamma \eta / \alpha \geq 0\) adjusted to the regularization parameter \(\alpha\), the Loewner ordering factor \(\gamma > 0\), and to the SFO uncertainty bound \(\eta \geq (\text{tr}(\Delta \Delta^+))\) (see [1, Sec. IV] for details). Under these conditions, the DEDR-optimal SO becomes

\[
F = (S^+ S + \alpha N_\Sigma A^{-1})^{-1} S^+
\]

i.e., a composition of the MSF operator \(S^+\) and the self-adjoint reconstruction operator \(K = (S^+ S + \alpha N_\Sigma A^{-1})^{-1}\). Next, following the notational conventions from the first paper of this series, we refer to [1, Sec. IV] for specifying the family of DEDR-related estimators for the considered there feasible adjustments of the processing-level degrees of freedom \(\{\alpha, N_\Sigma, A\}\)

\[
\hat{b}_{(p)} = F(p) YF(p)^+ (Y \Sigma^1 Y)^3\text{SK}_\text{diag}, \quad p = 1, \ldots, P
\]

where different employed SOs \(F(p)\) specify the corresponding DEDR-related estimators. In the case of the statistically optimal ML-motivated assignments of the processing-level degrees of freedom [1], i.e., \(\alpha = 1, A = D(b)\) and \(N_\Sigma = N_0 + \beta\), the optimal DEDR SO (11) depends nonlinearly on the desired SSP \(b\); hence, the nonlinear equation (12) must be next resolved with respect to \(b\), applying some properly designed POCS-regularized contractive iterative schemes as it was detailed in [1, Sec. VI].

### III. POCS-Regularized DEDR Techniques

To proceed from the general-form DEDR estimators (12) to the practically realizable SAR-adapted SSP reconstruction techniques, we follow the convex regularization paradigm invoked from the fundamental theorem of POCS [16]–[18]. Our approach incorporates the intrinsic factorization and sparseness properties of the SAR AFs [4], [14] into the construction of the POCS-regularized fixed-point iterative SSP reconstruction procedure that drastically reduces the overall computational load of the resulting algorithms.

#### A. Fixed-Point Iterations

To convert the general-form DEDR estimator (12) with the ML optimally specified degrees of freedom (i.e., \(\alpha = 1, A = D(b)\) and \(N_\Sigma = N_0 + \beta\)) to a POCS-regularized fixed-point iterative algorithm, we first define a sequence of estimates \(\{\hat{b}_{[i]}\}\) as

\[
\hat{b}_{[i+1]} = \mathcal{I}_{[i]} \left( \hat{b}_{[i]}; Y \right) = \mathcal{P} \left( \mathcal{K}_{[i]} S^+ Y \Sigma K_{[i]} \right)_\text{diag}
\]

\(i = 0, 1, \ldots\), where \(\mathcal{P}\) is a convergence enforcing projector (in our case, the POCS-regularizing operator constructed further on
in Section III)
\[ K[i] = K(\hat{b}[i]) = (\Psi + N_\Psi D^{-1}(\hat{b}[i]))^{-1} \]
represents the self-adjoint reconstruction operator at the \(i\)th iteration step, and
\[ \Psi = S^+ S \]
is the nominal system ambiguity operator (a \(K \times K\) matrix). Note that, to provide a convergent solution, the iterative operator \(T[i](\cdot)\) in (13) must be a contractive mapping \([16]–[18]\). Applying routinely the fixed-point technique \([16]\) to (13), we next construct the POCS-regularized matrix-form iterative SSP estimation algorithm
\[ \hat{b}[i+1] = \mathcal{P}\hat{b}[i] + \mathcal{P}T[i] \hat{b}[i], \quad i = 0, 1, \ldots \]  
(16)
The construction of this algorithm is detailed in Appendix A. Here, \(T[i] = T[i](\hat{b}[i])\) represents the solution-dependent matrix-form iteration operator defined by (A14) from Appendix A, i.e.,
\[ T[i] = 2\text{diag}(\{\Omega[i](\hat{b}[i])\}_\text{diag}) - H[i](\hat{b}[i]), \quad i = 0, 1, \ldots \]  
(17)
where
\[ \Omega[i] = \Omega[i](\hat{b}[i]) = I - \Psi - N_\Psi D^{-1}(\hat{b}[i]) \]  
(18)
\[ H[i] = H[i](\hat{b}[i]) = \Omega[i](\hat{b}[i]) \circ \Omega^*[i](\hat{b}[i]) \]  
(19)
\(\circ\) denotes the Shur–Hadamar (element-by-element) matrix product, and the zero-step iteration
\[ \hat{b}[0] = \hat{b}_{\text{MSF}} = \{S^+YS\}_\text{diag} \]  
(20)
is formed as an outcome of the MSF algorithm from the DEDR family (12) specified for the adjoint SFO SO \(S^+\) (see Appendix A for details).

The principal advantage of the fixed-point procedure (16)–(20) relates to the exclusion of the solution-dependent operator inversions (14), which are now performed in an indirect iterative fashion. This transforms the computationally extremely intensive general-form procedure (12) into the iterative technique (16)–(20) that is executable in polynomial time. To conceptualize the importance of the convergence enforcing POCS regularization, let us first evaluate the complexity of the constructed above fixed-point DEDR technique for a hypothetical computationally most expensive case when all matrix operations prescribed by (16)–(20) are performed with the full-format dense matrices. Assume that the arithmetic with individual matrix element has complexity \(O(1)\), consider the RS scene of a total dimension \(K = K_x \times K_y\), and assume that the fixed-point process is terminated after \(I\) iterations. By analyzing (16)–(20), one can deduce that, in this hypothetical case, the computational complexity of the constructed above fixed-point technique is on the order \(\sim O((K \times M^2) + (K^2 + K^3) \times I)\) for \(M = M_x \times M_y\) collected range–azimuth trajectory signal samples. For the real-world large-scale SAR imagery (referring to [4] and [5], the typical dimensions are \(K \sim 10^5, \ldots, 10^6\); \(M \sim 10^5, \ldots, 10^6\); and \(I \sim 10^2\), this complexity is too large to address (16)–(20) as a practical technique that is executable in (near) real computational time. The technically motivated way to decrease drastically the computational complexity is to benefit from the intrinsic factorization and sparseness properties of the fractional SAR AFs \([14], [21]\) via constructing the proper POCS-regularizing projector \(\mathcal{P}\) and incorporating it into the fixed-point procedure (16)–(20).

### B. SAR Model Specifications

To be specific, we adopt the model of a conventional strictly side-looking SAR with fractionally synthesized aperture as a sensor system. The SAR system geometry is illustrated in Fig. 1. The particular simulated sensor parameters are listed in Table I. Note that, in the case of maximum focused synthesized array, the regular trajectory signal must be approximated by the general Fresnel wavefield model \([5], [10]\). In the fractional synthesis scenarios (e.g., airborne fractional SAR system specified in Table I), the effective synthesized array is a fraction of the fully synthesized aperture that makes valid the spatially narrow-band second-order approximation of the Fresnel wavefield model \([2], [14], [21]\), in which case the regular trajectory unit signal in (1) is approximated by \([14]\)

\[ S(p; r) = S(t; \vartheta, \tau) \]
\[ = \exp \left[ -\frac{\pi(t - \vartheta - \tau/2)^2}{2T_s^2} \right] \sum_{i=-\infty}^{\infty} A(t - \tau - iT_0) \]
\[ \times \exp \left[ -\frac{2\pi v^2(t - \vartheta - \tau/2)^2}{\lambda_0} r_s \right]. \]  
(21)
This model is defined for a fractional SAR with a Gaussian directional diagram in the azimuth plane specified for the carrier that moves with constant velocity \( v \) along the nominal linear trajectory that is parallel to the \( x \)-axis at a constant height \( h \). The processing variables \( \vartheta \) and \( \tau \) are related to the Cartesian coordinates of the reference point \((x = v\vartheta, y = r_s \sin \varphi) = r \) on the scene plane \( r \in R \) as follows \([14]\):

\[
\vartheta = x/v = \text{time when the scattering element } r = (x, y) \text{ is on the traverse of the radar antenna, } (x = v\vartheta) \text{ (see Fig. 1);}
\]

\[
\tau = 2r_s/c = \text{delay related to the slant range } r_s = y/\sin \phi \text{ that is the distance to the scattering element at time } \vartheta, \text{ where } c \text{ is the velocity of light and } \phi \text{ is the probing angle.}
\]

In (21), \( A(t) \) is the complex envelope of the probe pulses, \( T_0 \) is the pulse repetition period, and \( T_s \) represents the length of the emission interval. In a hypothetical case of fully synthesized array \([2], [21]\), the maximum focused synthetic antenna length is \( L_{S\text{max}} \approx \lambda_0 r_s / \kappa_L \), where \( \lambda_0 \) is the wavelength of the radar signal transmitted and \( L_A \) is the horizontal aperture of the physical antenna. In the simulations, we adopted the traditional uniform model \([2], [21]\) of the antenna directional diagram \( G(y) \approx \text{const} \) for \( y \in (y_{\text{min}}, y_{\text{max}}) \) in the vertical plane that simply bounds the observation interval \((y_{\text{min}}, y_{\text{max}})\) and considered the case of a fractional synthesized aperture with the effective synthesized length \( L_\psi = \psi L_{S\text{max}} \), a function \( \psi \) of the maximum \( L_{S\text{max}} \). The angular spacing \( \Delta \theta \) between two scatterers \((x, r_s) \) and \((x', r_s)\) displaced at \( \Delta x \) along the \( x \)-axis in the same slant range plane \( r_s \) is expressed as \( \Delta \theta = \arctg(\Delta x / r_s) \); the slant range displacement delay \( \Delta \tau \) is expressed as \( \Delta \tau = 2 \Delta r_s / c \).

In theory \([2], [12], [14], [21]\), the resolution properties of a SAR system that employs the conventional MSF focusing are explicitly characterized by the AF of the unit signals \((21)\). The derivation of the AF

\[
\Psi(\Delta \tau, \Delta \theta) = [S(t; \tau, \theta), S(t; \tau', \theta')]_U = \int S(t; \tau, \theta) S^*(t; \tau', \theta') dt\]

\[
\Delta \tau = \tau - \tau', \quad \Delta \theta = \theta - \theta' \quad (22)
\]

of the SAR unit signals \((21)\) as a function of the delay–angle coordinates \((\tau, \theta)\) in the slant range plane was performed, for example, in \([14]\), and results in the approximation

\[
\Psi(\Delta \tau, \Delta \theta) = C \Psi_r(\Delta \tau) \Psi_A(\Delta \theta). \quad (23)
\]

Here, \( C \) is a normalizing constant (not essential in the simulations), \( \Psi_r(\Delta \tau) = [A(t - \tau), A(t - \tau')]_U \) represents the AF of the probe pulse signals as a function of the time delay variables in the slant range direction (a unimodal symmetrical function specified by the pulse modulation employed \([14], [21]\), and \( \Psi_A(\Delta \theta) \), given by the approximation \([14]\)

\[
\Psi_A(\Delta \theta) \approx \exp \left\{ -\frac{\pi(\Delta \theta)^2}{(\lambda_0 / 2L_\psi)^2} \right\} \quad (24)
\]

represents the AF over the azimuth angular coordinate \( \Delta \theta = \arctg(\Delta x / r_s) \) in the slant range plane. The corresponding azimuth AF \( \Psi_A(\Delta x) \) expressed over the Cartesian coordinate \( x \) in the ground scene plane \((x, y) = r \in R\) is given by

\[
\Psi_A(\Delta x) \approx \exp \left\{ -\frac{\pi(\Delta x)^2}{r_s^2(\lambda_0 / 2L_\psi)^2} \right\} \quad (25)
\]

Note that, in the conventional SAR processing \([2], [7], [21]\), to benefit from the range-angular AF factorization \((23)\), the images are formed (and postprocessed) in the slant range planes \((r_s, \theta)\); then, the resulting image is projected to the ground scene plane with the specified pixel spacing

\[
\Delta x = r_s \Delta \theta, \quad \Delta y = \frac{r_s'}{\sin \phi} - r_s / \sin \phi \quad (26)
\]

along the \( x \)- and \( y \)-axes, correspondingly related to the delay time \( \vartheta = x/v \) (see Fig. 1).

The delay–angular factorization of the AF \((23)\) implies that the DEDR-related algorithms can be executed independently over the delay and angular azimuth coordinates \((\tau, \theta)\) in each slant range gate \( r_s \). In the simulations, we considered the effective width \( \kappa_a \) of the azimuth AF, \( \kappa_a = \kappa_a(r_s, \theta) \), measured at the midrange \( r_{s0} \) (the reference midrange gate) in the SAR antenna footprint. To be specific, such \( \kappa_a \) is measured in pixels at the user-selected threshold (at 0.5 from the maximum value \( \Psi_a(0) \) at the midrange \( r_{s0} \)). For the adopted ordinary pixel representation format with the pixel width adjusted to the azimuth resolution cell corresponding to the maximum synthesized aperture length \( L_{S\text{max}} \), \([7], [21]\), the fractional factor \( \psi \) relates to the effective azimuth AF width \( \kappa_a \) as \( \psi = 1 / \kappa_a \). In the same manner, \( \kappa_r \) specifies the effective pixel width of the range AF in the slant range plane measured at the same user-selected 0.5 threshold of its maximum value \( \Psi_r(0) \).

Using the established AF factorization \((23)\), we next factorize the ambiguity operator \((15)\) over the azimuth–range scene frame. In the azimuth direction \( x \), the scaled composition \( S_x^T S_t \) defines the ambiguity matrix \( \Psi_t \) of the discretized model of the azimuth AF \((25)\), whereas, in the slant range direction \( r_s \), the composition \( S_x^T S_r \) corresponds to matrix \( \Psi_r \) that approximates the range AF. Due to the symmetry of the azimuth AF \( \Psi_A(\Delta x) \) and the range AF \( \Psi_r(\Delta r_s) \) in the slant range planes \((r_s, x)\), the corresponding matrices \( \Psi_x \) and \( \Psi_r \) have symmetric structures with finite pixel widths of the nonzero row/column strips (support pixel regions). The practical considerations regarding the adjustment of \( \kappa_a \) and \( \kappa_r \) to the employed SAR operational parameters and pixel representation scene format are discussed later on in Section VI. Now, we are ready to proceed with incorporating the AF factorization and sparseness properties into the POCS regularization.

### C. POCS-Regularized Fixed-Point DEDR Algorithm

To modify the fixed-point algorithm \((16)\) via enforcing the AF sparseness and factorization over the azimuth and range coordinates in the scene frame, consider the computations prescribed by \((16)\)–\((20)\) performed separately over the range pixels (i.e., along the \( y \)-axis) and over the azimuth pixels (i.e., along the \( x \)-direction). We formalize this stage by introducing
the range-azimuth factorization referred to as the factorization operator $P_{a \perp r}$ defined algorithmically as follows [26]: $P_{a \perp r}$ acts as a composition of the orthogonal $(a \perp r)$ sliding windows that zero the values outside the window apertures adjusted to the AF support regions $2k_a$ and $2k_r$, respectively. Second, following [16], [17], and [27] to enforce prior knowledge on the intrinsic positivity of the SSP (as power is always nonnegative), we impose, in addition to $P_{a \perp r}$, the positivity operator $P_+$ that has the effect of clipping off all the negative values, i.e., sets all the negative components equal to zero, thus enforces the image positivity constraint [27]. The factorization operator $P_{a \perp r}$ (with adjusted sliding support regions) and the positivity operator $P_+$ are projectors onto convex sets, i.e., POCS operators [16]; thus, a composition

$$P = P_+ P_{a \perp r}$$

(27)
defines POCS. While this definition in terms of projections sounds complicated, the algorithmic meaning of (27) is very simple. Acting on $b[i]$ (that may not be a member of the convex set at a particular iteration $i$), $P$ applied to $b[i]$ produces the member of the convex cone set that is composed of nonnegative elements that is nearest to $b[i]$ in the sense of minimization of the $L_2$ norm $\|Pb[i] - b[i]\|$ [16].

The application of $P$ constructed by (27) to the iteration process (16) yields the POCS-regularized fixed-point update rule

$$b_{i+1} = Pb[0] + P \{T[i]b[i]\}, \quad i = 0, 1, \ldots$$

(28)and this iteration process is enforced to converge to a point in the nonnegative convex solution set that is a direct sequence of the fundamental theorem of POCS [1, Sec. VI].

Now, we are ready to refer to the POCS-regularized DEDR fixed-point iteration procedure as follows.

Step 1) Choose the particular DEDR-related method from the family (12) specified by the criterion weight matrix $A$ and the regularization parameter $\alpha$. Specify the SFO uncertainty bound $\eta \geq (\text{tr}( \Delta \Delta^+))$, the initial observation noise power $N_0$, and the nonzero prior image gray level $b_0 > 0$. Compute the corresponding augmented noise power $N_\Sigma = N_0 + b_0 \eta$, and specify the POCS (27).

Step 2) Apply the MSF algorithm (20) to form the zero-step iteration, i.e., $b[0] = b_{\text{MSF}} = \{S^+ YS\}_{\text{diag}}$. The range-azimuth factorization $Pb[0]$ presumed by (28) allows performing such the MSF algorithm separately along the range y- and azimuth x-directions in the scene frame.

Step 3) Construct the fixed-point operator (17), and perform the iteration process (28) repeatedly for iterations $i = 1, 2, \ldots$

Step 4) Control the normalized updated discrepancy $\|b[i] - b_{[i-1]}\|^2/\|b_{\text{MSF}}\|^2$ at each iteration $i = 2, \ldots, I$, and proceed with the next iteration $i + 1$ until the solution (28) at some $I$th iteration satisfies the standard stopping rule condition [16]–[18], e.g., until the discrepancy $\|b[i] - b_{[i-1]}\|^2/\|b_{\text{MSF}}\|^2$ becomes less than some user-selected threshold. Consider the last iteration of (28) as the desired reconstructed discrete-form SSP $b = b[I]$.

IV. ADAPTIVE DEDR TECHNIQUE

The constructed above POCS-regularized DEDR-optimal technique requires proper adjustments of the corresponding degrees of freedom $(N_0, b_0, \eta)$ that specify the employed reconstruction operator (17). In the practical operational scenarios, the uncertainty bounding factor $\eta$ is usually unknown to the observer. Recall from [1] that, in practice, one cannot attribute the exact portion of the composite SFO perturbation term $\Delta$ to a particular source of disturbances. Moreover, modern airborne SAR systems are always equipped with a positioning system that allows knowing, with a given uncertainty, the trajectory deviations so that motion-compensation algorithms can be used at the initial image formation stage that employs the MSF focusing method [2], [21], [22]. Under these conditions, it may be questionable to assign accurately the bounding constant $\eta$ and the corresponding regularization factor $N_\Sigma$; using the uncertain prior model considerations. The proposed DEDR method can be therefore extremely useful as a way to account to the uncertain residual (i.e., uncompensated) trajectory deviation term, random SFO perturbations, and speckle noise in an adaptive fashion via adjusting the unknown regularization factor in the reconstruction operator (17) to its empirical estimate $N_\Sigma$ evaluated from the formed low-resolution speckle-corrupted MSF image (20). To proceed with such an adaptive extension of the DEDR method, here, we propose to employ the Lee local statistics method developed originally for SAR image despeckling [7].

A. Motivation: Local Statistics Method

The advantages of the celebrated Lee local statistics method [7] are that it does not require a statistical (correlation) model for the SAR image and multiplicative noise and performs despeckling at the image-postformation stage; thus, no prior statistical model of the raw SAR signals is required. Here, we conserve the notations from [7] to define the SAR pixelated image $\{z_k; k = 1, \ldots, K = K_x \times K_y\}$ formed applying the conventional MSF technique (20) that produces the corrupted image

$$z_k = b_{\text{MSF}}k = \xi_k i_k; \quad k = 1, \ldots, K = K_x \times K_y$$

(29)in which $\xi_k$ represents the composite multiplicative noise and $i_k$ corresponds to the hypothetical “texture” image pixel not affected by noise. Following [7], the mean and variance of the corrupted image (29) are estimated by the local mean $\bar{z}_k$ and local variance $\text{var}(z_k)$ in a $5 \times 5$ pixel sliding window; the mean texture image $i_k$ is evaluated via averaging the degraded image itself, i.e., attributing $\bar{z}_k = i_k$, and the texture image variance $\text{var}(i_k)$ is next computed by

$$\text{var}(i_k) = \frac{\text{var}(z_k) + \sigma_k^2}{\sigma_k^2 + 1} - i_k^2; \quad k = 1, \ldots, K = K_x \times K_y.$$

(30)
Under these considerations, the local statistics-based despeckling algorithm was constructed in [7, Sec. 3]

\[
\hat{\sigma}_k = \hat{i}_k + \frac{\text{var}(i_k)}{i_k^2 \sigma_z^2 + \text{var}(i_k)} (z_k - \hat{i}_k)
\]

(31)
to recover the texture image \( \{i_k\} \) from the formed speckle-corrupted MSF image (29), where \( \sigma_z \) represents the standard deviation of the image “flat” area evaluated in [7] for both single-look and multilook SAR processing modes.

B. Fusion of DEDR With Local Statistics Method

In the concise operator form, the adaptive despeckling algorithm (31) can be rewritten as

\[
\hat{i} = \mathcal{P}_L \hat{b}_{\text{MSF}}
\]

(32)
in which \( \mathcal{P}_L \) represents the Lee local statistics-based multiplicative noise suppression operator (algorithm) specified by the right-hand side in (31). Note that, in the convex regularization terms [17], [18], such \( \mathcal{P}_L \) defines POCS as it transforms the corrupted \( b_{\text{MSF}} \) into the despeckled texture image \( \hat{i} \) in the nonnegative convex solution set. Hence, the composition

\[
\mathcal{P}_\Sigma = \mathcal{P}_L \mathcal{P}
\]

(33)
defines a regularizing projector onto the convex set that can be employed in the DEDR procedure (28) instead of the original \( \mathcal{P} \) constructed by (27). This results in the following extended DEDR technique:

\[
\hat{b}_{[i+1]} = \mathcal{P}_\Sigma \hat{b}_{[0]} + \mathcal{P} \{ T_{[i]} \hat{b}_{[i]} \}, \quad i = 0, 1, \ldots
\]

(34)
with the fixed-point reconstruction operator \( T_{[i]} \) defined by (17), in which the unknown regularization factor \( \hat{N}_\Sigma \) can be evaluated directly from the speckle-corrupted MSF image (29) used to form the zero-step iteration \( \mathcal{P}_\Sigma \hat{b}_{[0]} = \mathcal{P}_\Sigma \hat{b}_{\text{MSF}} \) for the extended DEDR method (34). Following the Lee method [7], such empirical \( \hat{N}_\Sigma \) is computed via averaging the scaled estimates of the image local standard deviations over the scene frame

\[
\hat{N}_\Sigma = (1/K) \sum_{k=1}^{K} \left( \frac{\text{var}(z_k)}{\{\Psi\}_{kk}} \right).
\]

(35)

With these extensions, the previous warnings about (12) being ill conditioned do not apply since the whole purpose of the POCS regularization is to cure that same ill conditioning. Thus, the problem ill posedness is alleviated in an adaptive technically grounded fashion, namely, the following: 1) The factor that specifies the amount of regularization in the DEDR reconstruction operator (17) is now empirically evaluated by (35), and 2) the adaptive suppression of the multiplicative noise is incorporated into the extended DEDR technique (34) via fusing the POCS operators (33) that alleviate the ill conditioning of the overall SSP reconstruction inverse problem solved with such unified DEDR–POCS methods.

V. PERFORMANCE ISSUES

A. Convergence

The crucial practical issue is to enforce the convergence of the resulting fixed-point DEDR process (34) regardless of the zero-step initialization \( b_{[0]} \), i.e., regardless of the particular corrupted MSF image to be reconstructed/enhanced. We achieve this goal by the proper MSF-motivated construction of the convergence enforcing projectors \( \mathcal{P} = \mathcal{P}_+ \mathcal{P}_{a,l} \) and \( \mathcal{P}_\Sigma = \mathcal{P}_L \mathcal{P} \) as it is detailed in Appendix B, in which case the formal POCS procedure (B2) becomes the POCS-regularized fixed-point iterative DEDR algorithm specified previously by (34). Note that the employed range–azimuth factorization \( \mathcal{P}_{a,l} \) allows performing such the POCS-regularized fixed-point iterative technique separately along the range \( y \)- and azimuth \( x \)-directions in the scene frame \( R \). This results in the drastically decreased computational complexity of the overall POCS-regularized adaptive DEDR estimator that we next illustrate in the following practically motivated example.

B. Reduction of Computational Complexity

The benefit of the addressed POCS regularization consists in the drastically reduced computational complexity of the resulting iterative DEDR technique in the context of approaching the computational implementation of the algorithm (34) in (near) real time. Note that the concept of real-time computing is a user-specified category [17], [18]. On the one hand, the conventional RS users associate the real-time image formation and reconstructive processing with seconds or minutes (sometimes, with fractions of an hour), the time intervals corresponding to the acceptable evolution changes in the RS imagery [4], [22], [26], [33]. On the other hand, the users from the signal processing community usually restrict the real-time processing to the employed sensor signal modulation formats specified in fractions of seconds related to the conventional pulse repetition periods in the employed imaging radar systems [2], [3], [21]. In this study, we are oriented on the conventional RS users. In Section III-A, we evaluated the computational complexity of performing the DEDR method in a standard full-format iterative fashion to be on the order \( \sim O((K \times M^2) + (K^2 + K^3) \times I) \) for \( K = K_x \times K_y \) scene dimension, \( M = M_r \times M_a \) collected range–azimuth trajectory signal samples, and \( I \) executed fixed-point iterations. Consider the practically motivated example of the side-looking SAR from [6], [14], and [24] with the following operational specifications: \( K_x \sim 10^3 \); \( K_y \sim 10^3 \); \( M_r \sim 10^3 \); \( M_a \sim 10^3 \); and \( I \sim 30 \), . . . , 40. Then, the complexity of the full-format fixed-point iterative DEDR procedure is dominated by the order \( \sim O(10^{19}, \ldots, 10^{20}) \) that is definitely unacceptable for existing digital signal processing (DSP) platforms. Next, consider the SAR with the following typical sparseness frame specifications: \( \kappa_a \sim 10^1 \) and \( \kappa_r \sim 10^1 \) [2], [26]. Then, for the same MSF image (20) formed at the zero-step iteration, the adaptive POCS-regularized algorithm (34) executed sequentially over the range–azimuth pixel frames will require \( I \sim 20 \), . . . , 30 iterative runs involving \( K_{x} \sim 10^3 \) parallel multiplications, each one on the order \( \sim (\kappa_a \times K_x) \sim 10^4 \) performed over the azimuth coordinates in every of the
$K_y \sim 10^3$ range gates, followed by $K_y \sim 10^3$ parallel multiplications, each one on the order $\sim (\nu_y \times K_y) \sim 10^4$ performed over the corresponding azimuth frame coordinates at each iteration. This is a near real-time complexity procedure that is easy to implement in a parallel fashion with existing DSP platforms and even with high-speed personal computers. As an example, we refer to the simulations reported in Section VI that required approximately 11.93 s to execute the POCS-regularized fixed-point RASF algorithm (34) over the standard large-scale 1024 $\times$ 1024 pixel-format scene frame using a personal computer at 3 GHz with an Athlon 64 $\times$ 2 dual processor and 1.93 GB of RAM memory, a near-real time for conventional RS users [4], [22], [33].

C. Performance Metrics

For the purpose of objectively testing the performances of different DEDR-related SSP estimation algorithms, a quantitative evaluation of the improvement in the SSP estimates (gained due to applying the reconstructive SOs $\mathbf{F}^{(p)}$, $p = 2, \ldots$, from the DEDR family (12) instead of the robust MSF, i.e., the adjoint operator $\mathbf{F}^{(1)} = \mathbf{S}^+$) is to be accomplished. Because the resulting SSP is reconstructed over the rectangular $K_x \times K_y$ pixel-formatted frame, to be precise, we adopt here two quality metrics traditionally used in image restoration/enhancement [26], [28].

The first one is referred to as the improvement in the output signal-to-noise ratio (IOSNR) metric [26]

$$\text{IOSNR}^{(p)} = 10 \log_{10} \frac{\sum_{k=1}^{K} (\hat{b}_{\text{MSF}} - b_k)^2}{\sum_{k=1}^{K} (\hat{b}^{(p)}_k - b_k)^2}, \quad p = 2, 3, \ldots$$

(36)

where $b_k$ represents a value of the $k$th element (pixel) of the original SSP $b$, $\hat{b}_{\text{MSF}} = \hat{b}^{(1)}_k$ represents a value of the $k$th pixel of the rough SSP estimate $b_{\text{MSF}} = \hat{b}^{(1)} = \hat{b}_{[0]}$ formed using the conventional MSF method (20), and $\hat{b}^{(p)}_k$ represents a value of the $k$th pixel of the SSP reconstructed from the corrupted image $b_{\text{MSF}} = \hat{b}_{[0]}$, applying the $p$th particular technique ($p = 2, 3, \ldots$) from the DEDR family (12).

To evaluate the resolution enhancement achieved with the different reconstruction algorithms (12), we employ also the mean absolute error (MAE) metric [28]

$$\text{MAE}^{(p)} = 10 \log_{10} \left\{ (1/K) \sum_{k=1}^{K} |\hat{b}^{(p)}_k - b_k| \right\}; \quad p = 2, 3, \ldots$$

(37)

This metric is more suitable for quantification of fine-image reconstruction details, such as edge enhancement (sharpening) and resolution of small targets on the distributed scene [1], [28]. According to the quality metrics (36), (37), the higher is the IOSNR and the lower is the MAE, the better is the image enhancement/reconstruction with the particular employed algorithm from the DEDR family.

VI. Simulations and Discussions

A. Simulation Experiment Details

We simulated a conventional single-look SAR with fractionally synthesized aperture, i.e., the array was synthesized by the moving antenna as shown in Fig. 1. The factorized range–azimuth AF is composed of the conventional regular triangular range AF $\Psi_r(\Delta r_s)$ [2] and the Gaussian approximation (25) of the cross-range AF $\Psi_a(\Delta x)$. The corresponding AFs were projected to the ground scene plane with the pixel spacing specified by (26) along the $x$- and $y$-axes, correspondingly. As all the DEDR-related algorithms fall in the category of the postimage-formation techniques [14], [24] that perform image despeckling and enhancement via postprocessing the low-resolution speckle-corrupted MSF images, the simulations were performed at the image postprocessing level, i.e., avoiding the SAR raw signal simulations. The two tested scene images are shown in Figs. 2 and 3.

The first scene presented in Fig. 2 was artificially constructed and used for comparative evaluation of the resolution enhancement and despeckling performances of the tested DEDR-related algorithms. The second scene shown in Fig. 3 was borrowed from the real-world RS imagery [30] and was used as a...
hypothesised scene texture image to demonstrate and compare the performances of the different DEDR-related techniques in their application to the real-world large-scale RS scenes. Next, the image degradation and noising effects were incorporated to simulate the process of the formation of the degraded speckle-corrupted MSF images. First, following [21], the degradations in the spatial resolution due to the fractional aperture synthesis mode were simulated via blurring the original image with the range AF $\Psi_r(\Delta y)$ along the $y$-axis and with the azimuth AF $\Psi_a(\Delta x)$ along the $x$-axis, respectively. The degradations at the image-formation level due to the propagation uncertainties were simulated using the statistical model of a SAR image defocusing [11], [12], [26]. The fractional resolution along the $x$ and $y$ scene coordinates was controlled by assigning different effective pixel widths $\kappa_r$ and $\kappa_a$ of the range and the azimuth AFs and their varying over the scene that account to the range variation effect (26) and the uncompensated carrier trajectory deviations. Next, the blurred scene images were squared and degraded by the composite multiplicative (signal-dependent) noise because, technically, for a considered single-look SAR, the conventional MSF image formation algorithm (20) implies, first, the application of the regular adjoint SFO $S^+$ to the zero-mean Gaussian data realization $u$ and, second, performing the element-by-element (i.e., pixel-by-pixel) squared detection of $S^+ u$ to compose the corresponding pixel estimates $\{b_{MSFk} = |(S^+ u)_{kk}|^2; k = 1, \ldots, K\}$. Consequently, the MSF pixel estimates $\{b_{MSFk}\}$ are chi-squared distributed $\chi_2^2$ with two degrees of freedom, and such a distribution is a negative exponential Rayleigh distribution [7], [16]. Thus, to comply with the technically motivated MSF image formation scheme, the composite multiplicative noise was simulated as a realization of the $\chi_2^2$-distributed random variables with the pixel mean value assigned to the actual degraded scene image pixel that directly obeys the Lee model [7]. Such signal-dependent multiplicative image noise dominates the additive noise component in the data in the sense that $N_{\chi} \gg N_0$; hence, the estimate $\hat{N}_{\chi}$ performed empirically via (35) was used to adjust the regularization degrees of freedom in all the simulated DEDR-related SSP reconstruction procedures.

B. Algorithmic Specifications

We considered six DEDR-related estimators from the family (12), i.e., $P = 6$, renumbered now as $p = 1, \ldots, 6$. The first one is the conventional MSF estimator (20) that employs the adjoint SFO $F^{(1)} = S^+$. This degraded MSF image $\hat{b}^{(1)} = b_{MSF}$ was then postprocessed applying the Lee adaptive despeckling filter (32) that we refer to as the adaptively despeckled MSF image $\hat{b}^{(2)} = \hat{b} = P_{\hat{b}} b_{MSF}$. The nonconstrained robust spatial filtering (RSF) algorithm with the SO $F^{(3)} = F_{RSF}$ defined by [1, eq. (48)] was applied to enhance the original MSF image that corresponds to the incorrectly adjusted RSF method assuming no uncertainties in the data, $\beta = 0$, hence $N_{\chi}^{(3)}$ substituted by $N_0$ (in the simulations, the latter was assigned the value $N_0 = 0.01 b_0$ that corresponds to the prior SNR $b_0/N_0 = 20$ dB). The fourth simulated technique corresponds to the adaptive RSF method with the SO $F^{(4)} = F_{RSF}$, in which the unknown regularization factor $\hat{N}_{\chi}$ was empirically evaluated by (35) from...
the actual speckle-corrupted MSF image $\hat{b}^{(1)} = \hat{b}_{\text{MSF}}$. The fifth algorithm corresponds to the nonconstrained ASF method with the SO $F^{(5)} = F_{\text{ASF}}$ defined by [1, eq. (54)], i.e., the incorrectly adjusted RASF method assuming no uncertainties in the data ($\beta = 0$). Lastly, the sixth technique corresponds to the adaptive RASF method specified by the extended fixed-point DEDR algorithm (34) with the SO $F^{(6)} = F_{\text{RASF}}$ given by (7), in which the unknown regularization factor $\hat{N}_\Sigma$ in the fixed-point reconstruction operator (17) was empirically evaluated from the actual MSF image via (35) and the fused despeckling POCS operator (33) was employed at the zero-step iteration. Thus, the simulation experiment compares the five different enhancement techniques that range from the classical Lee adaptive despeckling method and nonadaptive DEDR-related RSF algorithm to the enhanced RASF technique (34) with fused despeckling and POCS regularization performed via (33). The simulations were run for two operational scenarios. The first one corresponds to the compensated defocusing errors (e.g., due to the employed autofocus [24]). In the second scenario, the system AF was additionally distorted over the azimuth frame within the realistic (worst case) azimuth AF width variation interval of $\pm 5\%$ to the nominal azimuth AF width. Such “extra” defocusing was performed to simulate possible aggregated uncontrolled SFO distortions (attributed to the disturbances due to the “heavy” propagation medium perturbations and uncompensated carrier trajectory deviations) that may occur in a much more severe operational scenario [5], [6]. For both scenarios, the simulations were run for different effective SAR fractional parameters $\kappa_r, \kappa_a$ as listed in Table I and different composite SNR $\mu_{\text{SAR}}$ defined as the ratio of the average signal component in the rough image formed using the MSF algorithm (20) to the relevant noise component in the same image $\mu_{\text{SAR}} = 10 \log_{10} \left\{ \frac{b_0}{N_\Sigma} \right\} \left\{ \text{tr} \left\{ \Psi_a \right\} \text{tr} \left\{ \Psi_r \right\} \right\}^{-1} \text{tr} \left\{ \left( \Psi_a \right)^2 \text{tr} \left\{ \left( \Psi_r \right)^2 \right\} \right\}$.

C. Simulation Results

In this section, we report the qualitative simulation results and the relevant quantitative performances evaluated via the two quality metrics (36) and (37) gained with five previously specified robust DEDR-related SSP estimators ($p = 2, \ldots, 6$). The simulation experiments were run for two examples of a SAR system characterized by different effective fractionally synthesized aperture frames $\kappa_a, \kappa_r$ (as specified in Table I) that, in different scenarios, operated under different SNR $\mu_{\text{SAR}}$.

Figs. 2 and 3 show the original scene images (artificially synthesized and borrowed from the high-resolution RS imagery [30], respectively) not observable with the simulated SAR systems. The images in Figs. 4–7 present the results of image formation and enhancement applying different DEDR-related estimators in different operational scenarios as specified in the figure captions. Figs. 4(a)–7(a) demonstrate the images formed applying the conventional MSF algorithm (20) in the uncertain scenarios. From these figures, one may easily observe that the MSF images suffer from imperfect spatial resolution due to the fractional aperture synthesis mode, the composite observation mismatches (in the second scenario), and are severely corrupted by the multiplicative signal-dependent noise due to the single-look SAR mode. In the first scenario, the degradations in
the resolution are moderate over the range direction ($\kappa_r = 3$) and much larger over the azimuth direction ($\kappa_a = 10$). In the second scenario, the fractional SAR system suffers from much more severe degradations because of the additional defocusing and lower spatial resolution in both directions ($\kappa_r = 6; \kappa_a = 16$).

Next, Figs. 4(b)–7(b) present the MSF images of the same scenes adaptively despeckled using the Lee local statistics algorithm (32). Figs. 4(c)–7(c) show the enhanced images formed applying the nonconstrained RSF algorithm. Figs. 4(d)–7(d) present the enhanced images formed using the constrained RSF adaptively adjusted to the particular uncertain scenario. The images enhanced with the ASF algorithm are shown in Figs. 4(e)–7(e), and the corresponding images optimally reconstructed using the RASF technique (34) are presented in Figs. 4(f)–7(f), respectively. Tables II–V report the quantitative performances evaluated via the two quality metrics (36) and (37) gained with five previously specified robust DEDR-related SSP estimators ($p = 2, \ldots, 6$).

From the reported simulation results, the advantage of the well-designed imaging experiments (POCS-regularized RSF, ASF, and adaptive RASF) over the case of badly designed experiment (nonrobust MSF, despeckling without DEDR enhancement, and imperfectly adjusted RSF) is evident for both scenarios. Due to the performed regularized inversions, the resolution was substantially improved (as reported also in Tables II–V)). The higher values of IOSNR, as well as the lower values of MAE, were obtained with the adaptive robust DEDR-related estimators, i.e., with the RASF technique empirically adapted to the uncertain scenarios. Note that IOSNR (36) is basically a square-type error metric; thus, it does not qualify quantitatively the “delicate” visual features in the reconstructed images; hence, small differences in the corresponding IOSNRs are reported in Tables II and III. Furthermore, the enhanced DEDR estimators manifest the higher IOSNRs and lower MAEs in the case of higher SNR. For the DEDR-optimal POCS-regularized RASF method (34), in addition, the ringing effect was substantially reduced. This is illustrated in Fig. 8, where the error images (i.e., the discrepancy between the actual corrupted noised scene image and the true multiple-target images) are exemplified for the second simulated scenario for three methods: 1) the multiple-target error image that is related to the conventional MSF image formation algorithm; 2) the corresponding error image observed after the application of the best existing Lee despeckling method; and 3) the corresponding error image that is related to the reconstructed scene image provided with the RASF algorithm (34), i.e., the best from the DEDR family. The difference in the perceptual image brightness representation format of the residual images in Fig. 8 is due to the adaptive rescaling performed by the corresponding DEDR-related algorithms in each particular case. The substantial reduction of the composite degradation factors, including the ringing effect, with the RASF algorithm is evident from the comparison of the error images in Fig. 8.

Next, in Fig. 9, we report the convergence rates (specified via the dynamics of the corresponding IOSNR and MAE metrics versus the number of iterations) evaluated for the most computationally consuming adaptive RASF technique (34); the curves relate to the simulated enhanced image reconstruction
Fig. 7. Simulation results for the second scenario with the second test scene. (a) Degraded SAR scene image formed applying the MSF method corrupted by composite noise (fractional SAR parameters: $\kappa_r = 6$ pixels, $\kappa_a = 16$ pixels, and SNR $\mu_{SAR} = 15$ dB). (b) Adapively despeckled MSF image. (c) Image reconstructed applying the nonconstrained RSF algorithm. (d) Image reconstructed with the constrained RSF algorithm. (e) Image reconstructed applying the nonconstrained ASF algorithm. (f) Image reconstructed applying the POCS-regularized adaptive RASF method.

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<th>$\text{IOSNR}^{(3)}$</th>
<th>$\text{IOSNR}^{(4)}$</th>
<th>$\text{IOSNR}^{(5)}$</th>
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TABLE II

IOSNR VALUES PROVIDED WITH FIVE SIMULATED DEDR-RELATED METHODS, $p = 2, 3, 4, 5, 6$.  
1 adaptive despeckling filter,  
2 nonconstrained RSF,  
3 constrained RSF,  
4 nonconstrained RASF, and  
5 POCS-regularized RASF results are reported for the first simulated scene and two simulation scenarios.

experiment exemplified for the two scenarios in Figs. 4(f) and 5(f), respectively, with the corresponding IOSNR and MAE metrics reported in Tables II and IV. These results are indicative of good convergence within approximately 25, . . . , 30 iterations of the fixed-point DEDR-optimal RASF algorithm in both the simulated scenarios. Some additional explanations regarding the convergence and numerical stability issues could be useful at this point. In the reported simulations, the iterations were run until the normalized IOSNR error metric approached the level at which the iterated procedure demonstrated asymptotical convergence but did not suffer from numerical instability. We observed somewhat different stability behavior of the iterated DEDR procedures in different performance metrics (IOSNR and MAE). Note that the DEDR method (by construction) is aimed at the constrained minimization of the squared-type balanced error metric (the minmax strategy specified by [1, eq. (34)]). Thus, a different error metric (e.g., MAE metric) which is not an objective function in the DEDR method may obviously demonstrate some numerical instability, i.e., increase after some number of iterations as illustrated in Fig. 9. This type of numerical instability is a subtle issue in constructing the regularized iterative techniques for different ill-conditioned problems, e.g., [16]–[18], [27], etc., because optimization (min, max, constrained optimization, balanced minmax, etc.) of the particular objective function may result in poorer quantitative performances evaluated by another quality metric. Furthermore, the relationship between the resulting IOSNR and MAE quality metrics and the visual reconstructed image quality is not
fully understood, although, of course, one would expect a high degree of correlation between the three. In our case, due to the POCS regularization, the appearance of the DEDR reconstructed images demonstrated a substantial improvement of up to 15 iterations from the MSF starting point. Next, the appearance of the reconstructed images changed very little from that of the 15 to 25 iterations. The changes became perceptually indistinguishable after 25, \ldots, 30 iterations. Moreover, the MAE quality metric in both the simulated scenarios demonstrated instability after 30 iterations (as illustrated in Fig. 9) that could be used as a practically inspired rule to terminate the iteration procedure. In the reported simulation experiments, after 30 iterations, the changes in the IOSNR metric were less than 4% and, after 40 iterations, less than 1% of the integral IOSNR improvements accumulated after the previous iterations, and the further continuation of the iterations did not improve the perceptual quality of the reconstructed images.

Lastly, we evaluated the required processing time for implementing the computationally most consuming RASF technique using the MATLAB 7.0 software in a personal computer running at 3 GHz with an Athlon 64 × 2 dual processor and 1.93 GB of RAM memory that was bounded by 11.93 s for 30 executed iterations. We do believe that, with special DSP computational platforms based on field programmable gate arrays [32], the computational time could next be substantially decreased via software–hardware codesign that employs the neural and systolic computational paradigms [18], [33], [34]. This paves a way for the (near) real-time implementation of the proposed POCS-regularized DEDR techniques in applications to the enhancement of the real-world large-scale RS imagery.
Fig. 8. Error images exemplified for the second simulated scenario for three methods. (a) Error image that is related to the conventional MSF image formation algorithm. (b) Corresponding error image observed after the application of the Lee despeckling method. (c) Corresponding error image that is related to the enhanced reconstructed scene image provided with the POCS-regularized RASF algorithm (34).

Fig. 9. Convergence rates of the RASF algorithm. (a) IOSNR metric versus the number of iterations. (b) MAE metric versus the number of iterations. (Curve A) First simulation scenario exemplified by the fine reconstructed image in Fig. 4(f). (Curve B) Second simulation scenario exemplified by the coarser reconstructed image in Fig. 5(f).

VII. CONCLUSION AND PERSPECTIVES

In this paper, we have proposed the POCS-regularized fixed-point iterative DEDR (DEDR–POCS) method particularly adapted for enhanced fractional SAR imaging in the uncertain RS environment. The unified DEDR–POCS approach leads to the SSP estimators that may be regarded as the adaptive postimage-formation enhancement procedures. We have presented the computationally efficient fixed-point iterative DEDR–POCS techniques with the corresponding computational recipes. To facilitate their application for the uncertain imaging scenarios, the adaptive scheme for the evaluation of the operational degree of freedom (regularization parameter) directly from the uncertain RS data has been proposed. We have demonstrated that, with the developed adaptive POCS-regularized DEDR procedures, the overall RS image enhancement performances can be improved if compared with those obtained using the conventional single-look SAR systems that employ autofocusing techniques or the previously proposed adaptive despeckling filters that do not unify the POCS regularization with the DEDR method. Therefore, the addressed unified DEDR–POCS approach puts, in a single optimization frame, adaptive SAR focusing, speckle reduction, and RS scene image enhancement in the uncertain environment.

In the future works, we would like to propose a more complicated regularization approach for speeding up the convergence of the fixed-point DEDR–POCS techniques realized via constructing the systolic computational schemes. In addition, we will focus on finding the different robust a priori statistical metrics for the randomized SSP models to incorporate those into the aggregated Bayesian-DEDR minimum risk strategy and expand such a framework for the RS image fusion problems.

APPENDIX A
DERIVATION OF THE ALGORITHM (16)

This section provides an explicit derivation of the POCS-regularized fixed-point DEDR estimation algorithm defined by (16) in the paper text. Here, we use the following notational conventions:

- \( \{ A \}_{\text{diag}} \) vector that is composed of the elements of the principal diagonal of the embraced square matrix \( A \);
- \( A \circ A^* \) square matrix formed via the Shur–Hadamard (element-by-element) product (\( \circ \)) of matrix \( A \) with its complex conjugate \( A^* \). Such product returns the matrix that is composed of the elements \( |A_{kj}|^2 \), i.e., squared modulus of \( A_{kj} \).

Furthermore, for a positive-valued \( b \), a diagonal matrix \( D = \text{diag}(b) \), and square self-adjoint \( A = A^+ \), the following equations hold:

\[
\{ AD \}_{\text{diag}} = \text{diag} \left( \{ A \}_{\text{diag}} \right) b \quad \text{(A1)}
\]
\[
\{ DA \}_{\text{diag}} = \text{diag} \left( \{ A \}_{\text{diag}} \right) b \quad \text{(A2)}
\]
\[
\{ ADA \}_{\text{diag}} = \{ ADA^+ \}_{\text{diag}} = (A \circ A^*)b. \quad \text{(A3)}
\]
Following the general fixed-point approach [16]–[18], we first specify the implicit fixed equation that must be satisfied at the solution to (12) for the ML optimally specified degrees of freedom (i.e., $\alpha = 1$, $A = D(\hat{b})$), i.e.,

$$\hat{b} = \{K(\hat{b})S^+YSK(\hat{b})\}_{\text{diag}} = \{KQK\}_{\text{diag}} \quad (A4)$$

where $Q = Q(Y) = S^+YS$ defines the measurement statistics independent on the solution $\hat{b}$ and $K = (\Psi + N_2D^{-1}(\hat{b}))^{-1}$ is the self-adjoint solution-dependent reconstruction operator that corresponds to (14) in the paper text. Multiplying (A4) by $\{Q(Y)\}_{\text{diag}}$ and using the properties (A1)–(A3), we replace (A4) by

$$\{(\Psi + N_2D^{-1}(\hat{b}))D(\hat{b})(\Psi + N_2D^{-1}(\hat{b}))^+\}_{\text{diag}} = \{Q(Y)\}_{\text{diag}}. \quad (A5)$$

Define next

$$\Omega = \Omega(\hat{b}) = I - \Psi - N_2D^{-1}(\hat{b}). \quad (A6)$$

With this definition, the previous equation (A5) can be rewritten as

$$\{(I - \Omega(\hat{b}))D(\hat{b})(I - \Omega(\hat{b}))^+\}_{\text{diag}} = \{Q(Y)\}_{\text{diag}} \quad (A7)$$

that, in the expanded form, is

$$\{(D(\hat{b}) - \Omega D(\hat{b}) - D(\hat{b})\Omega^+ + \Omega D(\hat{b})\Omega^+)\}_{\text{diag}} = \{Q(Y)\}_{\text{diag}} \quad (A8)$$

Using the properties (A1)–(A3), the nonlinear matrix equation (A8) is next transformed into the equation

$$\hat{b} = T(b; Y) \quad (A9)$$

that must be satisfied (iteratively approximated) on the desired estimate $\hat{b}$ with the right-hand side $T(b; Y)$ given by

$$T(b; Y) = 2\text{diag}\{\Omega(\hat{b})\}_{\text{diag}}\hat{b} + H(\hat{b})\hat{b} + \{Q(Y)\}_{\text{diag}} \quad (A10)$$

where

$$H(\hat{b}) = \Omega(\hat{b}) \circ \Omega^+(\hat{b}) \quad (A11)$$

To convert (A9) to an iterative algorithm, we next define a sequence of estimates $\{b[i]\}$ by a fixed-point iteration of the POC regularization form

$$b[i+1] = P b[i] + P T[i] b[i], \quad i = 0, 1, \ldots \quad (A12)$$

which incorporates a convergence enforcing projector $P$ (e.g., specified by the POCS regularization scheme constructed in Section III).

In (A12), the zero-step iteration

$$b[0] = \{Q(Y)\}_{\text{diag}} = \{S^+YS\}_{\text{diag}} \quad (A13)$$

is recognized to be the low-resolution image $b[0] = b_{\text{MSF}}$ formed using the MSF algorithm (20), and the matrix-form iteration operator $T[i] = T[i] b[i], \quad i = 0, 1, \ldots$, is defined by the first two terms at the right-hand side in (A10), i.e.,

$$T[i] b[i] = 2\text{diag}\{\Omega(\hat{b})\}_{\text{diag}} - H(\hat{b}) b[i]. \quad (A14)$$

Equations (A11)–(A14) specify the desired fixed-point DEDR algorithm for the SSP estimation that corresponds to (16)–(20) in the paper text.

**APPENDIX B**

**FORMALISM OF POC REGULARIZATION**

Following the POCS regularization formalism [16]–[18], the convergence enforcing projectors in the iterated procedure (28) are to be constructed as

$$P_\ell^i = I - \lambda_i (P_\ell - I); \quad P_1 = P_{a,r}; \quad P_2 = P_+; \quad P_3 = P_L \quad (B1)$$

where $\lambda_i, \ell = 1, 2, 3$, represent the relaxation (speeding-up) regularization parameters and $I$ is the identity operator. The iteration rule (28) for the composed projection operators (B1) becomes

$$b[i+1] = P_3 P_2 P_1 b[0] + P_2 P_1 T[i] b[i] \quad (B2)$$

$i = 0, 1, \ldots$, and is guaranteed to converge to the point in the intersection of the convex sets specified by $P_\ell^i$ provided $0 < \lambda_i < 2$ for all $\ell = 1, 2, 3$ regardless of the initialization $b[0]$ that is a direct sequence of the fundamental theorem of POCS [16, Sec. 15.4.5] (see also [1, Sec. VI] for the details). Note that the employed MSF-motivated specifications of the projectors in (B2), i.e., $P_1 = P_{a,r}$, $P_2 = P_+$, $P_3 = P_L$, with $\lambda_i = 1$ for all $\ell = 1, 2, 3$, and $b[0] = b_{\text{MSF}}$, satisfy these POCS convergence conditions, in which case the formal convergent POCS procedure (B2) becomes the POCS-regularized fixed-point iterative DEDR algorithm given by (34) in the paper text.

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